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THE DISCHARGE OF THE MISSISSIPPI RIVER.*

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READ NOVEMBER 6TH, 1895.

Difficulty of Ascertaining the Discharge of a Great River.—To ascertain with accuracy the discharge of a large river is at all times a delicate and difficult business. When the stream is of enormous dimensions, when the current is very swift and irregular, when the bed is very rough, the water full of boils, eddies and cross-currents, when the banks and bottom are highly unstable, and are suffering considerable mutations from day to day, then the difficulty of the task is so greatly magnified that engineers who have undertaken such a work have more than once been tempted to give up, in despair of securing even a reasonable approach to precision, and many have abandoned the effort and have been contented with approximations, not pretending to come nearer than within ten per cent. or so of the truth.

The Extreme High-Water Discharge of the Mississippi Has Never Been Measured.—It has never been found practicable to ascertain by direct observation the extreme high-water discharge of the Mississippi,

*The Discussion and Correspondence on this paper, part of which was presented at the meeting of November 20th, 1895, will be printed in a subsequent number.

because the stream has never been confined between its banks at such a stage. There are some portions of the alluvial valley of the Mississippi where the river has never been restrained by levees at all. In the part which is so controlled, the levee system is still so far from complete that in great freshets the levees have broken at one place or another before the top of the flood has been reached. The measurement of the discharge at such times has been only partial, and independent estimates have been made of the losses through crevasses, escapes over unleveed banks, etc., which estimates are usually very rough and unsatisfactory.

For a clear understanding of what is to follow, it will be necessary to have some knowledge of the topography of the Mississippi Valley.

Topography.—The alluvial plain of the Mississippi begins a short distance above Cairo, and embraces small portions of the Ohio Valley and that of the united upper Mississippi and Missouri Rivers. The Mississippi proper, or lower Mississippi, then, occupies a wide valley or bottom, lying between two nearly parallel ranges of tertiary hills, usually capped with loess. This plain runs in a direction from a little east of north to a little west of south, and occupies a distance, measured by parallels of latitude, of 7° 51′, or about 544 miles. By an air-line it is almost exactly the same. The course of the river from Cairo to the head of the passes is about 1 060 miles.

The river does not run through the middle of its valley, but like other streams skirts the hills, first on one side and then on the other. After leaving Cairo, its tendency seems to be southward, but it is held in check by the bluffs on the left bank, and consequently hugs their base, with the exception of a few small basins, till it reaches Memphis. The hills on the right or western bank are consequently far from the river. The valley here expands into a large area known as the Saint Francis Basin, because it is drained by the river of that name.

The Saint Francis Basin.—The bluffs which border the western limit of this territory meet the Mississippi at Helena, just below the mouth of the Saint Francis River. There are no gaps in this range of low hills (called Crowley's Ridge), and all the water which escapes over the banks of the Mississippi into the Saint Francis Basin returns into the main river, except such as is permanently retained in the swamps and lakes or is evaporated or lost by leakage.

There are several secondary ranges of hills in the Saint Francis

Basin, which play an important part in the phenomena of overflow. The first of these extends from Commerce to New Madrid, in Missouri. This ridge is so high and has so few gaps that it almost isolates the small part of the great basin cut off by it from the rest; the water that leaves the main river above New Madrid nearly all returns at or above that point.

The other ridges are only of local consequence.

The Saint Francis Basin exercises a great disturbing influence on the discharge of the Mississippi at high stages. A quantity estimated at 250 000 or 300 000 cu. ft. per second is abstracted from the river between New Madrid and Memphis, and returns, sooner or later, between Memphis and Helena. The basin is of great dimensions, containing about 6 000 sq. miles, exclusive of the New Madrid Basin, and its action as a reservoir is very complicated, and is still imperfectly understood.

The Yazoo Basin.—On the left bank, below Memphis and extending as far south as Vicksburg, is the great Yazoo Basin, formed by the Mississippi and a semi-circular range of hills. It is completely lined by levees, and exercises no disturbing influence except when crevasses occur. It is to be observed, however, that in 1882, 1883 and 1884, it was only partially protected by levees, and that enormous volumes of water escaped over the banks and through breaches in the dikes, returning at Vicksburg, where the Yazoo River enters the Mississippi, draining the entire basin. In 1890 several great crevasses occurred, by which the regimen of the river was considerably altered. The area of this territory is 6 648 square miles.*

The White River Basin.—Below Helena, on the right bank, there is the small but important White River Basin, containing 956 square miles. This area is formed by a recession of the hills to a distance of 13 or 14 miles. Through it enter the White and Arkansas Rivers, which interosculate at their mouths, and discharge mostly through a common orifice, the old mouth of White River, though the disused mouth of the Arkansas sometimes delivers a very considerable volume. The two mouths are about 9 miles apart, the mouth of the Arkansas being the lower. Several small streams contribute to drain this basin, and empty into the Mississippi above Arkansas City. The principal of these is Cypress Creek.

* These and the figures for the other basins are taken from the valuable publication, "The Mississippi from St. Louis to the Sea," by J. A. Ockerson, M. Am. Soc. C. E.

The White River Basin has no well defined natural boundary on its southern side, but is joined to the Tensas Basin. An artificial limit has been given to it, however, by the erection of a high and strong embankment along the right or southern border of Cypress Creek, extending to the uplands, which are here only about 5 miles in a straight line from the river. Though relatively high and pre-alluvial, these lands, like some of the ridges in the Saint Francis Basin, are not entirely above overflow. In times of great floods from the Arkansas and White Rivers, considerable volumes of water have been discharged around the end of the levee, as will be shown hereafter. During the past year, this levee has been enlarged and extended, and it will doubtless confine all floods of the Arkansas except the greatest.

The Tensas Basin.—The Tensas Basin is now also completely enclosed by levees. This area extends from the southern end of the White River Basin, or Cypress Creek, to the mouth of Red River. It contains 5 370 square miles. Its drainage is through various rivers and bayous into Red River.

The Atchafalaya and Pontchartrain Basins.—Below Red River is the Atchafalaya Basin, and on the left bank below Baton Rouge is the Pontchartrain Basin, neither of which is important to the present subject.

The configuration of the great alluvial plain and of its subdivisions, and the positions of the principal stations with which this paper is concerned are shown in Fig. 1.*

Gauges.—Gauges are placed at all points where tributaries enter, and at many important intermediate stations. They are placed at variable distances apart, averaging about 40 miles. Each of the important affluents also has one or more gauges along its course. Table No. I, on page 352, gives the distances of the gauge stations on the Mississippi from the Cairo gauge, and the elevations of their respective zeros. As the height of the mean Gulf level is not accurately determined, elevations are referred to the Cairo datum, as it is called, which is a point 290.84 ft. below the zero of the Cairo gauge. The Memphis datum, which is also much used in river work, is 13.13 ft. above the Cairo datum.

* Reproduced, by permission, from Mr. Ockerson's "The Mississippi River from St. Louis to the Sea," 1892.

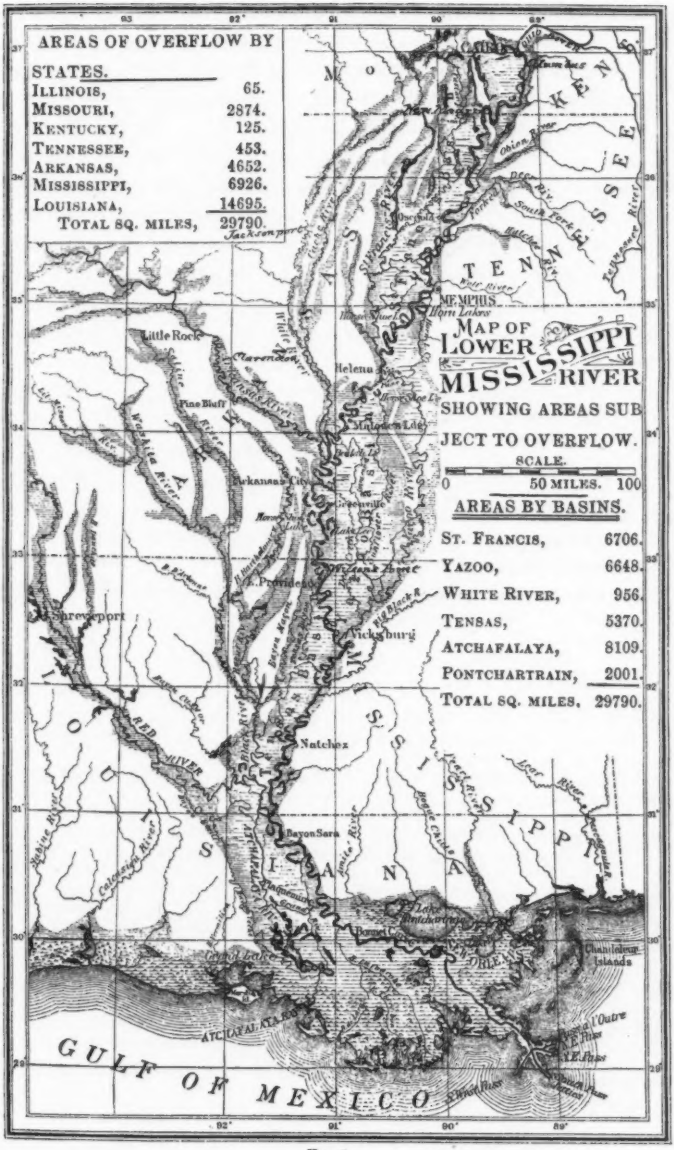
AREAS OF OVERFLOW BY STATES.
ILLINOIS, 65.
MISSOURI, 2874.
KENTUCKY, 125.
TENNESSEE, 453.
ARKANSAS, 4652.
MISSISSIPPI, 6926.
LOUISIANA, 14695.
TOTAL SQ. MILES, 29790.
MAP OF LOWER MISSISSIPPI RIVER
SHOWING AREAS SUBJECT TO OVERFLOW.
SCALE.
0 50 MILES. 100
AREAS BY BASINS.
ST. FRANCIS, 6706.
YAZOO, 6648.
WHITE RIVER, 956.
TENSAS, 5370.
ATCHAFALAYA, 8109.
PONTCHARTRAIN, 2001.
TOTAL SQ. MILES, 29790.
MEMPHIS
Little Rock
Helena
Pine Bluff
Arkansas Post
Vicksburg
Natchez
Bayou Sara
Shreveport
Lake Pontchartrain
Chandeleur Islands
GULF OF MEXICO
MISSISSIPPI
LOUISIANA
TENNESSEE
KENTUCKY
ARKANSAS

FIG. 1.

TABLE No. I.—Gauge Stations on the Mississippi River.

	Distance.	Elevation.
Cairo	00.0	290.84
Belmont (formerly Columbus)	21.3	287.14*
Morrison's Landing	69.0	275.80†
New Madrid	70.3	275.72
Cottonwood Point	123.0	250.62
Fulton	175.4	228.85
Memphis	230.0	203.97
Mhoon's Landing	277.3	181.48‡
Helena	306.5	161.98
Sunflower Landing	352.7	147.08
Mouth of White River	394.4	128.73§
Arkansas City	438.3	116.44‖
Greenville	478.3	108.00
Wilson's Point	531.2	93.45¶
Lake Providence	542.3	89.62
Vicksburg	599.3	66.04
Saint Joseph	648.7	52.74**
Natchez	700.3	36.89††
Red River Landing	765.1	23.85
Bayou Sara	799.8	23.95
Baton Rouge	833.3	20.06
Plaquemine	854.1	21.06
Donaldsonville	885.4	19.47‡‡
College Point	904.5	21.4
Carrollton	957.0	20.91
Fort Jackson	1 039.0	19.26§§

* This is the value formerly used. Levels of 1893 make it 287.394. The zero of the old Columbus gauge, used until June 8th, 1884, was 229.14.

† This gauge was used for New Madrid until 1882, when a new gauge was established at the latter place.

‡ The present gauge is nearly 2 miles below the former position. The change seems to have occurred at the beginning of 1892.

§ The distance is given by the Mississippi River Commission as 393.2. The gauge, however, is 1.2 miles below the mouth of the river.

‖ The elevation was given, prior to 1887, as 116.35.

¶ So for 1893. In previous years the distance is about the same, but the elevation was taken the same as at Lake Providence, namely, 89.62. This is not a regular gauge station, but is of importance from the number of discharges taken there.

** The distance is given by the Mississippi River Commission as 648.3. The gauge, however, is nearly half a mile below the town.

†† Since May, 1884, the distance of Natchez and points below has been shortened by about 12.5 miles by the cut-off at Cole's Point. This loss has been partially offset by increase of length in the bends due to caving banks, etc. The distances given in the table are the old ones, for want of accurate information as to the changes.

‡‡ Provisional.

§§ Since 1892.

The gauge at Little Rock on the Arkansas River is said to be 218 miles above Arkansas City. The elevation of the zero of the engineer's gauge is given by the Coast Survey as 241.50.

Jacksonport seems by the maps to be about 280 miles above Arkansas City on White River. The elevation of the zero of its gauge is not known. Neither is that of Clarendon on the same river. The latter place seems to be about 150 miles above Arkansas City.

Discharge Observations.—Discharges have been taken at Columbus, Point Pleasant, New Madrid, Fulton, Memphis, Hampton Landing (10 miles below Memphis), Helena, Arkansas City, Wilson's Point, Hays's Landing (10 miles below Lake Providence), Warrenton (8 miles below Vicksburg), Red River Landing and Carrollton; at Paducah, on the Ohio River; at St. Louis, on the Upper Mississippi, beside a number of isolated and scattered observations at other points. A few measurements have also been made on the lower tributaries, as at Clarendon, on the White River; at Little Rock, on the Arkansas, and at Alexandria, on Red River. A complete list, or what is intended to be such, of all discharges taken by the Mississippi River Commission or published by them is appended, and will be found in Table No. II.

TABLE No. II.—DISCHARGE OBSERVATIONS PUBLISHED BY THE MISSISSIPPI RIVER COMMISSION.

Place and date.	Where published.
Columbus, 1879	Report for 1894, p. 2849
Clarendon (White River), 1879	1894, p. 2855
Pine Bluff (Arkansas River), 1879	1894, p. 2852
Alexandria (Red River), 1879	1894, p. 2854
Hampton, 1879	1886, p. 2701
Fulton, 1879–80	1881, p. 98
Carrollton, 1879–80	1882, p. 112
Prescott, Wis. (Upper Mississippi), 1880–81	1882, p. 120
Winona, Minn., 1880–81	1882, p. 127
Clayton, Ia., 1880–81	1882, p. 133
Hannibal, Mo., 1880–81	1882, p. 138
Grafton, Ill., 1880–81	1882, p. 142
Saint Louis, Mo., 1880–81	1882, p. 147
Paducah (Ohio River), 1882	1883, p. 2584
Columbus, 1882	1883, p. 2607
Helena, 1882	1883, p. 2619

Place and date.	Where published.
Hays's Landing, 1882	Report for 1883, p. 2628
Red River Landing, 1882	1883, p. 2642
Bullerton, 1883	1889, p. 2649
Wilson's Point, 1883	1890, p. 3173
Carrollton, 1883, in Report for 1883, p. 2876, and	1889, p. 2642
Fulton, 1884	1889, p. 2648
Wilson's Point, 1884	1890, p. 3173
Hays's Landing, 1884	1891, p. 3488
Waterproof Cut-Off, 1884	1891, p. 3488
Point Pleasant, 1884–85	1887, p. 2818
Helena, 1884–85	1887, p. 2830
Arkansas City, 1884–85	1887, p. 2838
Warrenton, 1884–85	1887, p. 2843
Red River Landing, 1884–85	1887, p. 2852
Carrollton, 1884–85	1887, p. 2859
Wilson's Point, 1885	1890, p. 3174
Wilson's Point, 1887, in Report for 1887, p. 2888	1890, p. 3174 and 1891, p. 3496
Arkansas City, 1887, in Report for 1887, p. 2888	1890, p. 3164 and 1891, p. 3489
Wilson's Point, 1888, in Report for 1890, p. 3174, and 1891, p. 3496	
Wilson's Point, 1889, in Report for 1890, p. 3174, and 1891, p. 3496	
New Madrid, 1888–89, in Report for 1890, p. 3153, and 1891, p. 3501	
Helena, 1888–89, in Report for 1890, p. 3160, and	1891, p. 3508
Arkansas City, 1889, in Report for 1890, p. 3167, and 1891, p. 3492	
Warrenton, 1889	1891, p. 3513
Red River Landing, 1889	1891, p. 3518
Carrollton, 1889	1891, p. 3523
Below New Madrid, 1890, in Report for 1890, p. 3157, and 1891, p. 3531	
Plum Point Reach, 1890	1891, p. 3534
Memphis, 1890	1891, p. 3535
Helena, 1890	Report for 1891, p. 3536
Saint Francis Basin, 1890	1891, p. 3546
Arkansas City, 1890	1891, p. 3537
Wilson's Point, 1890, in Report for 1890, p. 3175, and 1891, p. 3527	
Wilson's Point, 1890 (low water)	1892, p. 3144
Warrenton, 1890	1891, p. 3538

Place and date.	Where published.
Natchez, 1890	Report for 1891, p. 3538
Red River Landing, 1890	1891, p. 3539
Baton Rouge, 1890	1891, p. 3538
Carrollton, 1890	1891, p. 3541
Cairo, 1891	1892, p. 3127
Columbus, 1891	1892, p. 3127
Plum Point Reach, 1891	1892, p. 3128
Fulton, 1891	1892, p. 3129
Memphis, 1891	1892, p. 3129
Helena, 1891	1892, p. 3130
Arkansas City, 1891	1892, p. 3132
Louisiana Bend, 1891	1892, p. 3135
Wilson's Point, 1891	1892, p. 3134
Natchez, 1891	1892, p. 3136
Red River Landing, 1891	1892, p. 3137
Carrollton, 1891	1892, p. 3138
Henry Clay Avenue (below Carrollton), 1891	1892, p. 3139
Saint Louis, 1892	1893, p. 3682
Columbus, 1892	1893, p. 3682
Fulton, 1892	1893, p. 3683
Memphis, 1892	1894, p. 2858
Helena, 1892	1893, p. 3684
Chicot City, 1892	1893, p. 3685
Arkansas City, 1892	1893, p. 3686
Wilson's Point, 1892	1893, p. 3688
Natchez, 1892	1893, p. 3690
Red River, 1892	1893, p. 3691
Carrollton, 1892	1893, p. 3693
Clarendon, 1892	1894, p. 2858
Little Rock, 1892	1893, p. 3694
Monroe (Ouachita River), 1892	1893, p. 3694
Alexandria (Red River), 1892	1893, p. 3695
Columbus, 1893	1894, p. 2825
New Madrid, 1893	1894, p. 2826
Fulton, 1893	1894, p. 2827
Helena, 1893	1894, p. 2828
Arkansas City, 1893	1894, p. 2829

Place and date.	Where published.
Wilson's Point, 1893	Report for 1894, p. 2833
Red River Landing, 1893	1894, p. 2837
Carrollton, 1893	1894, p. 2842
Little Rock, 1893	1894, p. 2840
Clarendon, 1893	1894, p. 2840

No discharges through outlets or crevasses have been included in this list.

History.

Humphreys and Abbot.—It is not now necessary to consider observations made at an early date. The discharges taken by Humphreys and Abbot were based on the assumption, supposed to be established by experiment, of an invariable datum cross-section. This having been carefully measured, the particular area corresponding to any gauge height was merely scaled off on the general plot, the area, therefore, being the same whenever the gauge showed the same reading. This assumption is now known to be erroneous. The area undergoes very great changes from day to day, most of which are accompanied by simultaneous changes of velocity. The measurement of the latter element, therefore, and not of the former, is likely to lead to very serious error.

The velocities were observed, in 1851, at as many different depths, and in as many different positions as possible. In 1858 they were taken at a uniform depth of 5 ft. below the surface. For the partial sections not adjacent to the shore, the observed velocities in 1851 were considered the mean velocities. In the shore divisions a coefficient of eight-tenths was applied for reduction.* In 1858 the observed velocities in the divisions not near the shore, at a depth of 5 ft., were reduced by means of a formula, half empirical, half rational, most laboriously worked out by a process, partly experimental, partly graphic, partly analytical, impossible to follow or to verify, consisting of combinations of curves, grouping of data and cutting and fitting generally. There results from all this hocus-pocus the following formidable expression:†

$$\frac{U_m}{U_5} = \frac{U_m}{U_m + \left(\frac{1}{3} + \frac{(0.317 + 0.06f)\,(10r - r^2 - 25)}{r^2}\right)(6v)^{\frac{1}{2}}}$$

* "Report upon the Physics and Hydraulics of the Mississippi River." By Humphreys and Abbot. Edition of 1876, pp. 234-35.

† The same, p. 276.

Coefficient of Reduction.—From this equation tables of corrections were prepared for the different observation stations, by which the observed velocities were finally reduced. The upshot of it all is that the ratio between the mean of all velocities in vertical planes, at a depth of 5 ft., and the mean of all the mean velocities in vertical planes, in calm weather, varies from 0.97 for low velocities to 0.99 for high. This result does not agree at all with later determinations. According to the elaborate measurements of 1882, the ratio, at all stages and stations, was about 0.91. The data supplied by Humphreys and Abbot, moreover, are very scanty, and are of no use for exact investigation. The results announced by them can be accepted, therefore, only as approximations.

The errors which appear in the work of these pioneers of experimental river hydraulics should not detract from their merits. When the entire novelty of the problem set before them is considered, the want of any precedents to guide them, the magnitude of the task, the imperfect means at their command, the short time allowed for the work and the rapidity with which it was performed and the results matured and announced, their achievements seem little short of herculean. That they made mistakes is not surprising. There was as much of the soldier as of the engineer in General Humphreys, and his combative and positive character has given a deep tinge to all his opinions. Nevertheless, the work must always stand as a monument of original research, of colossal labor and of engineering genius. All the work done since has been based on the model set by them. Engineers of the present day owe them an everlasting debt of gratitude. Used with discrimination, their work is still of great value.

Low-Water Board of 1879.—The present system of gauges was organized in 1871, and the records extend back to the latter part of that year.

In 1879, a number of discharges were taken on the Mississippi and some of its tributaries by Colonel Suter and Colonel Benyaurd. The results of these have only lately been made public. They have frequently been made use of by Colonel Suter and others as the basis for calculations. They are generally known as the work of the "Low-Water Board" of 1879.

The Mississippi River Commission, 1879-80.—In 1879 the Mississippi River Commission was organized; and in the fall of that year, under

the direction of the secretary, Captain Leach, discharges were taken at Fulton and at Carrollton, and observations for slope, cross-section, etc., were made at Lake Providence. These were by far the most comprehensive and accurate series of gaugings yet made. They occupied a whole year, and covered a very wide field, embracing the determination of curves of vertical velocities, transportation of sediment, measurement and travel of sand waves, variations of slope and cross-section, etc.

Current and Sediment Observations of 1880-81.—In 1880-81 a series of observations was taken on the upper Mississippi from Prescott, Wis., to St. Louis, now most useful for its careful data on the subject of the transportation of sediment in suspension.

Observations of 1882.—These works were a preparation for the yet more important labors of 1882. In this year occurred the greatest flood of which there is any authentic or accurate record. Parties were early placed in the field. Discharges were taken at Paducah, on the Ohio River, just above Cairo; at Columbus, Helena, Hays's Landing and Red River Landing on the Mississippi. At the same time, measures were taken for the determination of the escape over the banks into the Saint Francis Basin, and the unprotected parts of the White River, Yazoo and Tensas Basins, as well as through the numerous breaks in the levees of all the basins which occurred during this portentous flood. A mass of information was thus gathered which, imperfect as it is, furnishes most of the existing knowledge of the maximum flow of the Mississippi, and has contributed greatly to the correct understanding of the general movements of that mighty stream.

The necessity for great accuracy in the measurements of the discharge was not at first apparent. Had it been never so evident, the means were not at hand for its attainment. The men were comparatively untrained, the methods new, the outfits wholly inadequate, scraped together in a hurry, and eked out often with makeshifts and home-made devices, adapted to the work by the genius or non-genius of the engineer in charge. The current meter was at that time a comparatively untried instrument, and the form in use was not suited to the turbulent, muddy, drift-laden Mississippi. Existing formulas for the use of floats were often inapplicable to the new conditions. New sources of error were discovered. Base lines were wrongly

measured in the hurry. Lead-lines had stretched several feet before the error was detected. Instruments were broken by rough usage and could not be replaced, or sometimes failed to do the work expected of them. Notwithstanding all these drawbacks, the measurements are not extremely discordant with one another or with the probabilities of the case, and indeed sometimes exhibit fewer discrepancies than others taken with more precautions and refinements. The records, as they are presented, appear in their native nakedness. No attempts have been made to reconcile their discrepancies. Indeed, they greatly need an intelligent discussion, such as has been bestowed on those of 1884-85. The notes which accompany the reports are usually very meagre, and give very little insight into special sources of error which were undoubtedly present.

In 1883 and 1884 measurements were continued of the escapes of water through crevasses and outlets and over banks.

Observations of 1884–85.—The next noteworthy series of observations was taken in 1884–85, extending from about October to April. Complete measurements were taken at Point Pleasant, Helena, Arkansas City, Hays's Landing, Warrenton (below Vicksburg), Red River Landing and Carrollton. These observations were published in 1887, with a full discussion and very satisfactory notes by L. L. Wheeler, M. Am. Soc. C. E. Mr. Wheeler's comments throw much light on the difficulties which attend the taking and the reduction of discharge observations, and point out the means of correcting many errors.

Observations of 1889.—From the failure of appropriations very little was done in the way of gauging the flow of the river until 1889, when a series of measurements covering four or five months was made at New Madrid, Helena, Arkansas City, Warrenton, Red River Landing and Carrollton. Full details are given of methods, etc.; 1889 was a year of low water, and the observations embrace only medium stages.

High-Water Discharges of 1890, 1891, 1892 and 1893.—The years 1890, 1891, 1892 and 1893 were all high-w ter years, and discharges were taken at several of the stations during the period of flood, and sometimes isolated measurements were made at low water. Many of the observations were taken with extreme care, and in almost all cases the data are accompanied by complete explanatory notes. Sometimes very curious phenomena are revealed by these high-water observa-

tions, and some of the apparent anomalies which they present have not yet been satisfactorily explained.

Measurement of Cross-Sections.

It is not intended to give an essay on the practice of gauging rivers, which would of itself require an elaborate paper, and would demand for its treatment much experience on the part of the author. It is only desired to call attention to the sources of the many errors and discrepancies in the records of this sort of work, and especially in the case of the Mississippi, with the view of discriminating between real errors and mere apparent anomalies or puzzling incidents, which may be very genuine and may afford the key to the discovery of new laws.

Soundings.—Depths have been measured from the earliest times in very nearly the same way. In Humphreys and Abbot's day, in 1879–80, and generally up to a recent date, soundings have usually been taken by the "drifting" method, the line being lowered to a position near the bottom just before passing the range line, and dropped to actual contact at the moment of crossing. From a skiff, catamaran or launch, doubtless this is the best way. Where a steamboat is used, it is customary, at the present time, to proceed as in taking deep-sea soundings on shipboard, that is, the line is stretched fore and aft, along the guards of the boat, the lead being in front. The lead is then heaved, and the sounding is read as the line comes to a perpendicular at the after part of the boat. Either method is good enough if due pains are used to insure accuracy.

The first, most obvious and most necessary precaution to be taken is to see that the lead-line is of the correct length. In the careful observations of the present day, the line is tested before and after each day's work and the necessary correction inserted in the field-book opposite each sounding. The notes accompanying the earlier observations give very few particulars in this regard. Humphreys and Abbot used a welded chain. The common lead-line has been employed by almost all others, however, often checked by the occasional use of piano wire. Almost no assurances are given in the work of 1882 that any pains were taken to insure accuracy of measurement in depths. In Mr. Wheeler's discussion of the observations of 1884–85 there are many instances showing the possibilities of error that exist in taking

soundings, and likewise the frequency of such errors. At Helena, on October 11th, the lead-line was tested and was found to have shrunk 3 ft. in 50 ft., and there were no data for determining what the error was on previous days. At Arkansas City the corrections were made once a week, "but on some occasions large changes took place in the lead-line with no means of determining just when the change occurred." At Warrenton the soundings were corrected for errors of the lead-line. The corrections during the latter part of the season were very large, being about 12 ft. in 70 ft. At Red River Landing no tests of the lead-line are recorded in the notes, and no corrections are given or have been applied. Mr. Wheeler notes that a large change took place in the datum area between January 3d and January 6th. "It is much to be doubted if such a change really took place. The only note which has any relation to the subject is a statement in the journal that a new lead-line was tagged January 5th. It is also probable that a heavier lead came into use on January 6th." The change thus recorded was about 10 000 sq. ft. of area, corresponding to 2.4 ft. of mean depth.

The location of the sounding stations is also of great importance. If the exact position of the point where each sounding is taken is uncertain, how can one day's observations be compared with another's? The usual practice has been, first to establish a range line by proper signals or targets, and then to locate velocity stations by the intersection of oblique lines, each defined by two numbered targets, on one or the other bank. Soundings are often taken at the velocity stations, but not necessarily; but as the measurements of depth are taken at much shorter intervals than the measurements of velocity (50 ft. as compared with 200 ft. or more), the intermediate stations have been fixed by angles taken from a base line by a transit on shore or a sextant from the boat. To show the possibilities of error in this particular, reference is again made to Mr. Wheeler's discussion. At Point Pleasant soundings were located both by signals and by angles.

"The locations by angles and by signals in the main are quite discrepant, and it has been found impossible to reconcile the two methods of location. The notes of the location have not been recorded or the computations of the same preserved, so that it is now impossible to prove that the signals were in the right places to make intersections 200 ft. apart. On the other hand, the sextant angles show discrepancies among themselves, so that it is certain that the sextant locations are erroneous. There is also some evidence that the signal locations

are erroneous, but as all velocity measurements and all soundings after January 1st, 1885, were located by these signals alone, it was decided to reject the sextant angles entirely and depend upon the signals for location."

In the early part of the work some very large changes took place in datum areas, and in such a manner as to place in doubt the measurements of areas at that time.

At Warrenton, positions were located with a pocket sextant, too small an instrument to give accurate locations with the small angle employed. The soundings were not taken at regular intervals, and up to December 15th, the velocities were also measured at irregular distances.

At Red River Landing, the locations of the soundings were determined both by signals and by sextant angles. The two locations do not agree exactly, but the difference is small.

At Carrollton, the soundings were taken at irregular intervals, and were located by the series of signals on the right bank. In the revision many differences were encountered. Some of them could be settled by a reference to the original notes, but such as could not were settled arbitrarily. In some few instances the notes have been corrected arbitrarily where they were evidently largely in error. "On a large number of days velocities were observed, but no soundings were taken. There seems to have been no good reason for this. It was decided to interpolate depths for those days when soundings were not taken. In this manner depths have been interpolated for 39 dates."

Degree of Accuracy Attainable.—As a measure of the accuracy attainable in these locations, at Wilson's Point, in 1891, soundings were located simultaneously by a transit on shore with a base of 1 600 ft. on the Louisiana shore, and by a sextant on the boat to a base of 1 000 ft. on each bank. The difference varied from 0 to 39 ft., the latter, however, being exceptional. The discrepancy generally did not exceed 6 ft.; and the mean difference of 94 sets, including the extreme case, was 2.3 ft.

It is the opinion of the engineers in personal charge of this work that it is possible to measure the cross-section with a near approach to accuracy; and that the figures given in the reports of discharge observations as representing the several areas are, in the main, nearly correct.

It is to be remembered, however, that a change in method or a change in the personal constitution of the party may very easily cause an apparent change in area. For instance, the substitution of a new leadsman may make an immediate apparent scour or fill where none really existed. The change of a single foot in a series of soundings will make a change of 4 000 sq. ft. or so of area; and a change of a foot in depths of 60 to 100 ft. is easily made.

Changes of Cross-Section.—Frequently very large changes of cross-section occur from day to day, being generally afterwards compensated by a change or changes, sudden or gradual, in the opposite direction. It is the opinion of the observers that these changes are usually genuine. They may be due to the passage of sand waves, to violent caving or filling, or to various causes, some of which are yet unknown or obscure. In corroboration of the position that these changes are real, it is to be noted that the violent fluctuations in question are usually accompanied by as sudden a change, in the inverse sense, of velocity; which would not be the case if the discrepancies were fictitious and proceeded from errors of measurement—unless by an occasional coincidence.

Measurement of Velocities.

The current meter is used almost to the exclusion of all other means on the Mississippi, at the present time, other methods being employed only occasionally as checks or under exceptional circumstances. Many observations of importance, however, have been taken by floats; and in view of certain objections which experience has shown to be attached to the current meter, engineers are sometimes found who are disposed to recur to former methods.

Floats.—The systems of measurements of velocity by double floats and by rod floats are well understood. In the former, there is no essential difference at the present day from the method pursued by Humphreys and Abbot. In the latter, there are some modifications in the Mississippi practice, owing to the great depths existing. Rod floats cannot be successfully handled if more than 35 or 40 ft. long.* Consequently, in very deep water, such as is habitually found throughout the greater part of the cross-section in the Mississippi in time of flood, the float does not reach half way to the bottom.

* Mr. J. H. Davis, in "Report of the Mississippi River Commission for 1883," pp. 2601, 2616.

It does not, therefore, fulfil its mission of determining the mean velocity in each vertical by direct observation, but the results have to be reduced by a formula. It has, to be sure, some advantages over a surface float or one immersed only 5 ft., because it is influenced by a greater number of filaments, and thus many accidental and temporary irregularities are eliminated.

As to the confidence to be placed in the results furnished by these instruments, the testimony of the engineers who have made actual use of them in work on the Mississippi may be taken. Says Captain Leach, at Carrollton, in 1879–80:*

"In a series of double floats run in rapid succession, at the same depth and over the same path, the individual results have been observed to vary by as much as 25 per cent. The velocity of a single float may, by accident, be the average velocity for that portion of the stream, but in general it will not; and however exactly it may be measured, it cannot be accepted as more than an approximation to the quantity desired. Add to this the error in determining the cross-section, and the uncertainty as to whether the section measured is the really effective one, and it seems improbable that these discharges can be relied on within 10 per cent."

At Paducah, in 1882, W. G. Price, M. Am. Soc. C. E., says:†

"I think double floats do not give the mid-depth velocity, owing to the effect the current has on the upper float and connecting wire. The upper float is also affected by the wind. For this reason the measurement would be too great except where there is a strong up-stream wind, when it might be too small. Measurements of vertical curves of velocity with double floats in an up-stream wind show a much greater decrease of velocity near the surface than the same measurements with the current meter.

"The mid-depth velocity is not always a mean velocity. I think it varies with different sections and with different stages of water, being from 1 to 4 per cent.

"There is no check on any of the observers, one of whom being a careless man will spoil the accuracy of the work. The shore observers may work with their instruments out of adjustment; they may miss the float, but give the signal as though all was right; or the floats, to save labor, may not be adjusted to mid-depth, and the chief of the party may not know it.

"With rod floats, when they are observed from the shore, as with double floats, there is no check on any of the observers. Careless observers will let the float project too much out of water, so that it

* "Report of the Mississippi River Commission for 1882," p. 101.

† "Report of the Mississippi River Commission for 1883," p. 2584.

is affected by the wind. In deep water floats often do not reach to mid-depth.

"As the vertical curve of velocity varies very much with different sections, and also at different points on the same section, it is impossible to make a formula which will reduce the observed velocity to a true mean. For this reason, when the floats do not reach nearly to the bottom, they may not be as accurate as the double floats.

FIG. 2.

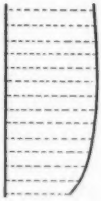

FIG. 3.

"During the time these measurements were taken the river fell from about a medium stage to low water. At first the rod float discharges were the greater, but when the river had fallen much lower, they were less than those taken with the meter. When the rod float discharges were the greater, the vertical curves of velocity were like Fig. 2, and when they were less, they were like Fig. 3. This showed that the observed velocity of the floats had not been reduced enough at first, and afterwards too much, though the same formula (Francis's) was used all the time."

Mr. J. H. Davis, at Columbus, in 1882, remarks:*

"I was led to believe that at times the surface and subsurface parts were acted upon by almost opposite forces. At times the surface float would be suddenly drawn to right or left, in which case it would be considerably inclined and partially immersed. Sometimes it would incline backward or forward. In each of these cases the surface float had a powerful influence in retarding or accelerating the sub-float; the velocity of the former not representing the correct velocity of the water in the vicinity of the latter. The above indications were frequently noticeable in high water. Sometimes also in high water the sub-float was apparently held up, the assumed immersion not being reached. This was shown by the fact that several feet of wire were sometimes taken in before there was any appreciable weight of the sub-float. By an examination of the vertical velocity observations at Fulton, it will be found that in the sets in which two or more floats were run at the same depth the velocities differ from 1 to 15 per cent."

Current Meters.—With current meters, very much depends on the kind of instrument employed, its suitability to the particular class of work it is called upon to do, its mechanical excellence, the care exercised in keeping it properly adjusted, its manipulation, its rating and the accuracy with which its indications are recorded and the results

* "Report of the Mississippi River Commission for 1883," p. 2616.

computed. It is a delicate instrument, with several bearings and an electrical connection, and its ratings are found to vary considerably with even slight changes in repair or maintenance.

The first forms of current meter which were devised were of the windmill, screw or propeller pattern; that is, the vanes which were moved by the current revolved in a plane perpendicular to the direction of the latter. Such was the design of the original Woltmann meter, of the Révy meter which was used on the Paraná, Uruguay and La Plata Rivers in 1871, of the Harlacher meter and of several others. The instruments used in the Mississippi work are constructed on a different principle. They are of the "anemometer" type, as it is sometimes called; that is, the vanes are replaced by cups on the ends of arms which revolve in a plane parallel to the thread of the current. The motive force here is the difference between the pressure exerted by moving water on a concave and on a convex surface. The meters used in the early experimental work on the Mississippi were of the type devised by General Ellis for use on the Connecticut River in 1877, and were of the anemometer pattern. The changes which the experience of 1882 and of subsequent years showed to be desirable in the Ellis meter have resulted in the evolution of the Price meter, which is now in general use. Of late, there has been a disposition to revise the decision made in favor of the anemometer type of meter, and to return to the screw plan, as in the new Ritchie and Haskell meter; so the question cannot be considered settled.

There are at least two sources of error in the use of any kind of meter. First, the instruments or the observers may not tell the truth. Second, they may record and transmit with perfect accuracy what has happened; yet this recital of facts may not be at all what is sought, although apparently so. The meter may deliver with absolute truthfulness its tale of so many revolutions in so many minutes, yet in attempting to translate these into feet per second grievous blunders may be made, more especially when it is assumed that this travel of a thread of water was in a direction parallel to the banks of the river and to the surface of the stream; and still more particularly when it is accepted as a fact that this solitary observation represents the mean velocity of a large portion of a swift and turbulent river; the interval between the observation stations being never less than 150 ft. and frequently as much as 300 ft., and the depth often from 50 to 100 ft.

A great part of the knowledge of the flow of the river rests upon the testimony of these mute and unintelligent witnesses. It is believed that they tell their story with nearly perfect fidelity, as far as they go. There is a great deal that they do not tell. It is the engineer's part to interpret their deliverances correctly, to divine or to ascertain from other sources the information which the meters fail to give, and generally to supply their deficiencies.

The first point to be considered is the mechanical perfection of the instrument. Old and worn meters will not give correct indications; they will register the number of revolutions probably, but this number will not correspond to a definite velocity in feet.

Secondly, the meter must be maintained in perfect order, the bearings and connections kept in a uniform state as far as possible, and, every time a change of any kind is made, the instrument should be rerated. It is the experience of the observers that when a meter is taken apart and cleaned, its rate changes; that when a new electrical connection is put in, a bearing oiled, or, in short, the slightest alteration made, the effect is immediately perceptible and is often considerable.

Rating of Meters.—Assuming the instrument to be in perfect condition, it is essential that it should be correctly rated. It is thought by many observers that here lies the *nodus* of the difficulty. The operation is simple enough in principle. It consists merely in dragging the meter through still water at a rate determined by observation. The method now approved is to lower the instrument from a skiff to a distance of 5 ft. or more in a pool of considerable depth, and then have the skiff towed by a rope between fixed range lines, taking the time by a stop watch. Mr. Charles H. Miller, an observer of ability and experience, declares * that the skiff should never be pulled with oars, and that the observers should be cautioned about keeping the same positions and not swinging their bodies about during the observation. "In a rating taken from a steamer drawing 2.5 to 3 ft. of water, with the meter in 10 ft. of water, a large error was found over other ratings of the same meter, and it was then rated at 20 ft. under the surface." Mr. Miller thinks a slight current or even an oscillation not objectionable if reasonably constant, care being taken to get the observations in pairs, and to have the times of transit the same both ways.

* See *Engineering News*, January 24th, 1895.

Ratings in shallow water are not to be relied on, nor where the meter is brought too near to the skiff or steamboat.

These matters were not understood at first, and it is only by patient observation and practice that the importance of extreme care in rating has become known. It is thought by some skilled engineers that many of the discrepancies hitherto unaccounted for are due to careless or unskilful rating.

A meter wheel revolves under the impulse of moving water, and against the resistance of friction. The object of rating is to ascertain the ratio between a revolution of the wheel and the velocity of the current. If the wheel makes one revolution in a second, while the water moves 2 ft., then the ratio is 2 to 1. If there were no friction this ratio would be constant, and three revolutions would correspond to a velocity of 6 ft. The same result would follow if the resistance of friction were proportional to the velocity, and the equation for rating the meter would be $y = a\,x$; where y is the velocity sought, x the number of revolutions in a second, and a a constant expressing the relation between them. The friction of solids, however, is supposed to be nearly independent of the velocity. The resistance of the bearings will therefore be nearly the same at low velocities as at high, and there will be a greater proportionate resistance in the former case. Therefore, there will be fewer revolutions of the wheel in proportion to the velocity. One revolution of the wheel will no longer represent 2 ft. of velocity, but, perhaps, 3 ft. Therefore, an expression is desired which shall show this varying relation. General Ellis * gave as this equation the formula $y = a\,x + b$, in which b is the term expressive of the constant resistance of friction. The same equation was adopted by Mr. Price and was used in the reductions of 1882 and of several subsequent years; in fact, until very recently.

There are several elements, however, which enter implicitly into this expression and influence the values of a and b and prevent them from being constants. The quantity a, so far as it depends upon the theoretical rate of travel of the perimeter of the wheel, cannot be regarded as constant, for there is the "slip" to be considered, the law of which is unknown, and the unequal action of the filaments upon the arms of the wheel at different angles, accelerating in one direction and retarding in another. The term b, so far as it depends upon the fric-

* See "Report of the Chief of Engineers for 1878," Vol. II., p. 308.

tion of the bearings, cannot be constant, for it will be influenced by the perfection of the machinery, the proper lubrication of the same, etc., all of which have been shown to be subject to serious irregularities.

The friction of the bearings and working parts is not the only resistance to be considered, for the action of the water against the convex surface of the cups or vanes, which are moving contrary to the current, is also important. The motion of the meter, indeed, depends upon the difference between the forces exerted by moving water upon cup-shaped and wedge-shaped surfaces. These forces are governed by laws altogether different from those of solid friction, for they depend on the square of the velocity. How important a part is

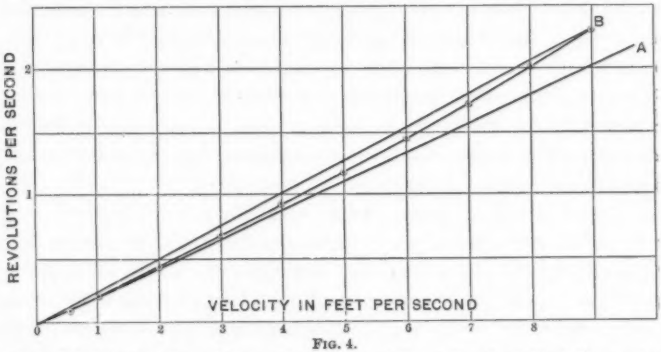

FIG. 4.

played by the total resistance is apparent from the fact that the meter will not register very low velocities at all.

These reasons seem to be sufficient to explain the fact that the relation between the revolutions of the meter and the velocity is not at all a simple one, and that the formula expressing that relation is not the equation of a straight line. It approximates to it, upon a casual inspection, so that the line, when plotted from actual experiment, upon a large scale, is not easily distinguishable from a straight line by the eye. It is not surprising, then, that the earlier observations were reduced upon that supposition. The constants were ascertained by combining a number of equations of condition, derived from observation, on the principle of least squares.

The error introduced by this assumption is very material. In a given instance, taken from actual practice,* if a tangent, $O\,A$, be

*At Arkansas City, May 18th-23d, 1893. The observations were taken with all possible care, with a new and well tested meter, by Mr. Thomas C. J. Baily, Jr., U. S. Surveyor, and with every precaution suggested by previous difficulties.

drawn to the lower part of the curve *O B*, it will diverge from the curve (see Fig. 4) at 6 ft. by 0.5 ft., at 7 ft. by nearly 0.7 ft., at 8 ft. by more than 0.9 ft. If a chord, *O B*, be drawn from 0 to 9 ft. it will depart in the middle of the curve by about 0.4 ft. If a straight line be drawn so as to coincide, as nearly as may be, with the curve, it will depart from the curve at several points from 0.1 to 0.2 ft., sometimes in excess, sometimes in deficiency. The low velocities will be too high, the medium velocities too low, the high velocities too high, and the percentage of error will be very different for the several cases. In the low velocities it may be 10 or 20%; in the high, only 1 or 2 per cent. It is obvious that by such a method, while the actual error at each point might not be very great, yet it might be sufficient to obscure all traces of the law governing the progression of the discharge, intricate as this is, and influenced by numerous delicate perturbations. When the circumstances of the observations, the accuracy of the instrument, its condition, the skill, experience, accuracy, judgment and good faith of the observers are unknown, it is not easy to assign the limits of probable error in rating meters.

Present Practice in Rating Meters.—It is the rule at present to rate meters frequently, and to apply the ratings directly to the observations taken during the adjacent period. This has only lately been the practice. In the older observations, notably in 1884–85, meters were rated only occasionally, perhaps once in a season, and apparently with little care or method. In 1882, so far as the record shows, the meters were rated frequently, and there is no reason to suppose that the methods employed were less accurate than those of 1884–85. Indeed, in some respects, they were perhaps better. The meters, however, were mostly too light and delicate for the work, the methods were novel, and most of the observers without experience.

Nowadays, from the rating observations a line is plotted, the revolutions being the ordinates, and the velocities the abscissas. Fig. 4, just referred to, is an example of this work. From the line thus obtained, the number of revolutions being given, the velocity may be taken by inspection. In short, every precaution is taken that the instruments and the men shall tell the truth. Whatever errors occur are mostly due to difficulties or faults in the method. The difficulties are mostly of two classes—difficulties of manipulation, or execution, and difficulties of situation. Supposing these to be overcome, there are still faults in the method.

Difficulties of Manipulation.—In a stream like the Mississippi, and especially at high water, it is almost or quite impracticable to take discharges from an anchored boat, and, indeed, it is doubtful whether an anchored boat could be kept much more quiet than a boat under steam; therefore the boat is always kept under way. The object is to hold it on the range line, and as nearly as possible at the velocity station, while the observation is in progress. With all possible skill in management, there will be a little drift, direct and lateral. Care is always taken, however, to see that the boat is on the range line at the beginning and the end of the observation, so that the excursions up and down stream may compensate one another. Lack of this precaution has often caused serious errors. It is the belief of good observers that lateral drift may be kept within narrow bounds, and that the error should be inconsiderable.

It is not always practicable in swift currents and at great depths to hold the meter in a horizontal position. If it is tilted at an angle, the bearings come at places where it was not designed that they should, and in old instruments especially the rate is greatly influenced thereby.

Winds and waves do not influence the mean velocity, but they shift the axis of the curve of vertical velocities, and instead of being at nearly 0.6 of the depth, as usual, the place of mean velocity in the vertical may be at 0.1 or at 0.8. Again, the waves produce a rocking or an up-and-down motion of the boat that is communicated to the meter, and the latter registers these movements as well as the velocity of the water. Large perturbations of the discharge have been produced in this way.

Difficulties of Situation.—With all the precautions that experience has suggested, there are sometimes discrepancies or anomalies in discharge observations so great that they cannot be accounted for in any of the ways heretofore indicated. Instructive examples of this are to be found in the observations taken in 1891 and 1892 at Arkansas City and at Wilson's Point. Up to this time it had been impossible to check the observations at one point by those taken at another, owing to the intervention of tributaries, which introduced a disturbing element difficult to calculate. Thus Helena could not be checked by Arkansas City in 1884–85, because the White and Arkansas Rivers poured out between the two stations a variable and unmeasured volume, which sometimes amounted to 300 000 cu. ft. per second. In 1891,

however, C. McD. Townsend, M. Am. Soc. C. E., being dissatisfied like everybody else with the existing state of the knowledge of discharges, had simultaneous gaugings taken at the two places above named, 93 miles apart, and without any tributary or other disturbing condition between. The results showed the discharge at Arkansas City to be 150 000 to 200 000 ft. more than at Wilson's Point. Had the difference been only for a day or two it might have been explained, well or ill; but it was continuous and persistent. At Arkansas City, from March 28th to April 4th, the records were 1 390 586, 1 419 422, 1 423 730, 1 408 553, 1 419 321, 1 405 641. At Wilson's Point they were 1 272 440, 1 288 000, 1 219 436, 1 200 578, 1 247 445, 1 275 904, 1 234 182. Averages, 1 411 209 and 1 248 283. There was no crevasse or other disturbing influence between them. The same discrepancy kept up till April 23d, after which date the difference slowly diminished. This unexpected result put the engineers to their wits' end. The meters were compared and exchanged. They had been several years in service, and their bearings were somewhat worn. They were rated frequently, and the rates of the same meter at different times were found to be very discrepant.

Captain Townsend considered the meters to be unreliable, and ordered two new ones. The next spring, 1892, a still greater flood developed, and high-water discharges were taken at the same two places with the new instruments. The results were more discrepant than before. The discharges at Arkansas City from April 26th to May 9th averaged 1 598 682; at Wilson's Point, 1 364 277. The instruments were compared, side by side, and meters and observers were changed, without affecting the result. There were no disturbing influences between the two places except the overflow of the foreshores outside of the levees, which increased only 1.7 ft. during the interval of 13 days.

Several natural explanations offered themselves, but none were adequate to explain so great a discrepancy. The reservoir space of the whole area included between levees from Arkansas City to Wilson's Point is only about 228 miles. An excess of discharge at the upper station of 200 000 cu. ft. per second would fill the whole reservoir 2.7 ft. deep in one day. The actual increase of depth was only 0.1 or 0.2 ft. The loss of such a volume by evaporation or percolation was out of the question. Besides, there was no general loss of volume of discharge coming down stream, as shown at other stations.

Obliquity of Current.—A plausible solution of the difficulty was suggested, it is believed, by General Comstock, and Captain Townsend is disposed to accept it. This attributes the excessive registration at Arkansas City to obliquity of current. It is known that in bends there is a lateral circulation of currents from bank to bank,* the upper current being centrifugal, the lower centripetal. So in "crossings," at the reversion points of bends, the current leaves the one bank to occupy the other. The level of the water on the left bank is sometimes as much as 1 ft. above the level of the water at a point directly opposite on the right, or the reverse. This difference of head produces a strong lateral movement, which, acting in conjunction with the regular down-stream flow, has a resultant acting at a considerable angle with the axis of the stream. Captain Townsend had float observations made in 1891 for determining the direction of the current. At Arkansas City the average deflection was about 10° 30′, indicating a discharge, as observed with meters, about 1.5% too great. At Wilson's Point, also, subsurface floats were run for the same purpose. The result showed a deviation

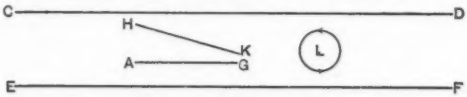

FIG. 5.

even greater than at Arkansas City, namely, about 12°, indicating an excess in measurement of about 2 per cent.

Vertical Currents.—It is likely that vertical currents also exert a powerful effect. What is demanded of the meter is that it shall give the velocity of a filament in each vertical which runs parallel to the general direction of the bank and to the bottom, or, rather, surface. What the meter gives is the velocity of the filament in the direction in which the latter is moving at the time, be that direction direct, oblique, horizontal, inclined or vertical. If Fig. 5 be taken to represent a river in plan, CD and EF being the banks, then what is sought is the velocity in the direction GA. What the meter gives is perhaps the velocity in the direction KH. If the sketch, however, be taken to represent the profile of the river, of which CD is the water surface and EF the bottom, then the velocity is sought in the direction GA in a horizontal plane. It is obtained perhaps in the inclined

* See "Encyclopædia Britannica," vol. xii, pp. 498–9, article, "Hydromechanics."

direction KH. If the inclination was known in each case, the projection of KH on GA, which would be the quantity sought, could be obtained easily. Generally the inclination is unknown. In time of flood, when discharge observations are most important and are generally taken, a depth of 0.6 implies a submersion of 40 ft. or more, where the movements of the meter cannot be observed, and can be only partially controlled.

The general direction of the current at the surface or at any desired depth may be ascertained with more or less accuracy by the use of floats. This is a step in the right direction. But what is wanted is the particular position of the meter, both in horizontal and vertical planes, during the time of observation at each station. A considerable uncertainty must always be felt as to the indications of ordinary meters, especially at the period of high water, even though the general obliquity of the current be known, from the great irregularity and turbulency of the current at such times. Should the meter, at the time of registration, encounter an eddy, in a horizontal plane, or a "boil" in a vertical plane, as at L in Fig. 5, the indicated velocity and direction may be valueless, unless both velocity and direction be taken at the same moment of time. Several ingenious devices have been proposed for this purpose, and others are under projection. The "direction current meter" of Messrs. Ritchie and Haskell* is an instrument by which the lateral deviation of the meter is ascertained by means of a submerged compass, the travel of the needle of which is recorded on a dial above water by an electrical connection. It is not known what measure of success has been attained.

Other Disturbing Influences.—There are a number of disturbing influences, which, like the above, may be classified under the head of "difficulties of situation," which affect the velocity at any station from time to time, and render any registration which occupies only a few moments liable to error of greater or less amount. Some of them have been referred to. All are imperfectly known, when known at all, and doubtless there are others which are yet unsuspected. A competent and experienced observer,† stationed at Arkansas City in 1893, remarks:

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* Described in *Engineering News*, January 10th, 1895.

† Mr. T. C. J. Baily, Jr., in "Report of the Mississippi River Commission for 1894," p. 2815.

"When the river is in a normal condition, the discharge varies but slightly within a short time; but when near its limits, immense fluctuations occur within small intervals. This will be better appreciated by an inspection of the velocity curves. These show that when the river is very high, the immense flats above will flow out and fill in within a very few minutes of each other, affecting the flow by at least 100 000 cu. ft. per second."

The same engineer, when stationed at Wilson's Point, in 1892, reports:*

"Two thousand feet above the range on the Mississippi side, the Duncansby Chute pours an immense volume of water into the river at a high velocity, the axis of the chute forming an angle of 39° with that of the river at the point of entrance. It is believed that this water, by eating out the bar which has formerly existed at the east end of the section, is one of the causes of the great variations which may be observed from day to day in the discharges. Stations 2, 3 and 4 were the ones affected, the current varying on them so greatly from minute to minute that it was extremely difficult to judge when a true average velocity had been obtained. For this reason, these stations were occupied from 8 to 20 minutes. * * * From June 6th until the end, the current on Station 1 was running up stream as determined by double floats, and the entire partial discharge of this section thereafter was subtracted from the sum of the remaining partial discharges. The position of the seam between the up and down stream currents was not at all constant, it shifting from a point 50 ft. outside of Station 1 to Station 2 within a minute."

Discharges taken at Chicot City, a few miles above Arkansas City, in 1892, showed differences of 100 000 ft. from those taken at the latter point, and this sometimes in excess, sometimes in deficiency.

It is thus seen that, to obtain correct discharges, or discharges approximating correctness, especially at high water, much depends on the skill, experience, conscientiousness and judgment of the observer. Even with the utmost skill and care, a discharge observation made in the ordinary way is simply the determination of the velocities prevailing at the moment at 10 or 15 or at most 20 isolated points distributed throughout a cross-section of 200 000 sq. ft. or more, together with the sounding of the section at intervals of 50 ft. or so. That results obtained in this way should not accurately represent the true discharge of the river is not surprising. If they did represent the true discharge for the time being, it might be very far from the mean discharge for the whole day.

* "Report of the Mississippi River Commission for 1893," p. 3669.

The "Flanking Method."—To eliminate, as far as possible, the effect of lateral obliquity of current, and to obtain a more nearly true mean of the velocity of the numerous fillets of water from the one side of the river to the other, Captain Townsend has tried a method which he calls the "flanking method," which consists in starting from one side of the river with the meter submerged to a certain depth, usually 20 ft., and slowly "flanking," or moving across, keeping the steamboat's head up stream, and keeping on the discharge range, having the record running continuously, but noting the registrations and the time as each intersecting range is passed. This operation is repeated at once in the reverse direction, the two trips constituting one complete observation.* Of course the results are corrected for the sidewise motion of the boat.

This method was tried, in conjunction with the ordinary plan, in 1893, at both Arkansas City and Wilson's Point, and was found to afford a valuable check. The results of the two trips from bank to bank usually show the existence of a lateral current, though not a very strong one, at the depth of immersion of the meter, the difference of velocities going and coming seldom exceeding 0.2 ft. and generally being less than one-tenth. They are more uniform than the results obtained by the old plan, and have occasionally corrected the excessive figures of the latter. One day, for instance, at Arkansas City, the discharge found by the old or stationary method was 1 667 306; by the flanking method, 1 566 054. On the whole, the two tally more closely than might have been expected.

In 1893, the discharges at Arkansas City and Wilson's Point agree fairly well; and when checked by the results of the flanking observations, they present a very coherent mass of data, affording the most trustworthy measurement of the extreme high-water discharge ever obtained on the Mississippi. They show, among other things, that there is no considerable loss of volume between the two stations by evaporation or leakage, and that between two stations so closely adjoining, with a nearly stationary river, there is but little weight to be given to the reservoir effect of the channel and the overflowed bank.

Possible Error in Deficiency.—The discharges of 1891 point to the possibility of an error in deficiency. From the probabilities of the

* Report of Mr. Charles H. Miller on observations at Wilson's Point, in "Report of the Mississippi River Commission for 1894," p. 2816.

case it would seem that the Arkansas City discharges were nearer to the truth than those observed at Wilson's Point. That a meter should register too high a velocity, from cross-currents, etc., is credible enough; but how can it register too low a velocity? Only from incorrect rating, one would suppose; yet the ratings of this particular instrument are remarkably accordant; indeed it was a newer and better meter than the one used at Arkansas City. If the recorded discharge at Wilson's Point is indeed too small, it is due to some cause yet unknown. Perhaps an eddy section was subtracted in the belief that the eddy was general when it was only superficial. Perhaps the reading of the meter at a submersion of 0.6 differed from the mean velocity in the vertical by a greater proportion than usual. Whatever the cause, it was general, for the error, if it be an error, is persistent.

Faults of Method.—Whatever method of determining velocities be used, its correctness depends upon an assumed relation between the velocities at different depths in vertical planes and the mean velocity in the same planes. By the method of double floats, and also by the ordinary or stationary method of meter observations, the velocity is taken at a depth which is a certain fraction of the total depth, usually 0.5 or 0.6. By rod floats, in great depths, the velocity of the mean of a certain number of filaments to a depth of 35 or 40 ft. is taken, and reduced by a formula depending on the curve of vertical velocities. In the flanking method the meter is kept at a constant depth below the surface, which depth bears, of course, a perpetually varying ratio to the total depth; so different factors are used at the different proportional depths to reduce to the mean of the vertical.

It was announced by Humphreys and Abbot, as one of their discoveries,* "that the ratio of the mid-depth velocity to the mean velocity in any vertical plane is practically independent of the depth and the width of the stream, of the mean velocity of the river, of the mean velocity of the vertical curve, and of the locus of its maximum velocity. In other words, it is a sensibly constant quantity for practical purposes." Their conclusion, supported by the figures which they bring to bear, is that at depths varying from 86 to 27 ft. the ratio has an extreme range only from 0.9868 to 0.9798.

In 1882, the parties who took the discharge observations made a large number of velocity measurements in vertical planes, which are

* "Report upon the Physics and Hydraulics of the Mississippi River," p. 311.

published in the "Report of the Mississippi River Commission for 1884," pages 2691–2723, tabulated for each tenth of depth from surface to bottom. The observations are more accordant than might have been expected from the difficulties of such work, with new men and insufficient appliances and at great depths. A few of the figures presented seemed to be anomalous, from misprints, errors or simply from unusual conditions.

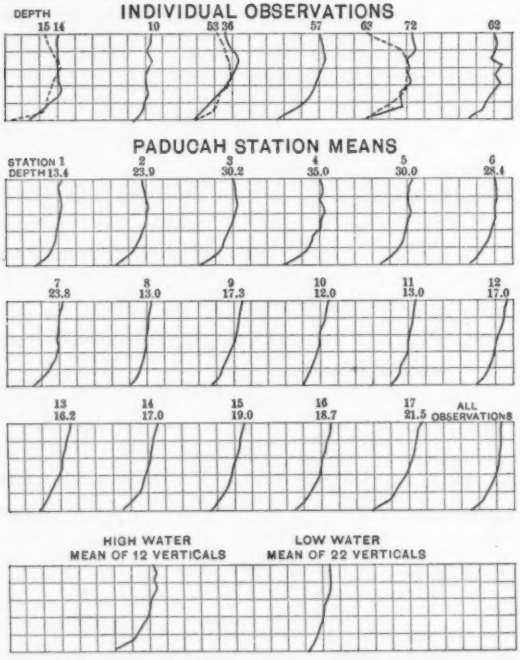


FIG. 6.

Plotting the Curves of Vertical Velocities.—In using these observations, the curves may be plotted which are developed by the velocities at different depths at the several stations, namely, Paducah, Columbus, Helena, Hays's Landing, and Red River Landing. In order to compare the curves with one another, the convenient practice of the engineers of the Mississippi River Commission may be adopted, and the depths expressed as fractions of the whole depth and the velocities as

fractions, proper or improper, of some standard velocity. The criterion generally adopted is the mid-depth velocity, partly because it is convenient, partly because it has been supposed to bear a close and nearly constant relation to the mean velocity.

When single observations are plotted, the curves are very irregular. They are not curves at all, in fact but broken lines. A few specimens are given in Fig. 6.

It would be a perplexing and, perhaps, an unprofitable task to attempt to unravel these intricacies. It would be better to take the means of different groups of these data, and try to eliminate in this

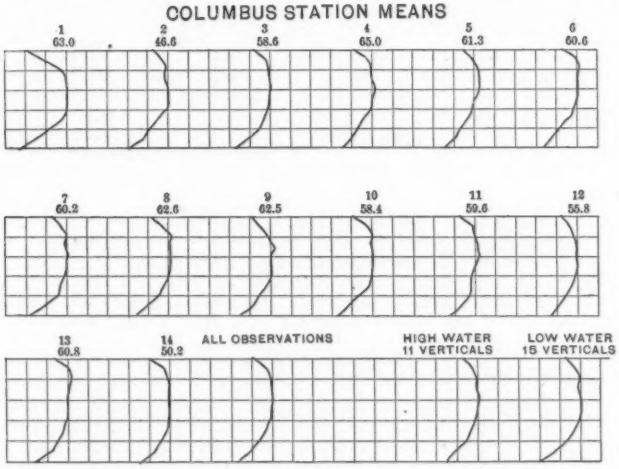


FIG. 7.

way accidental and local disturbances. The data of 1882 contain the "station means"; that is, the means of all the observations taken in each vertical plane throughout the season. At each of the five observation points a range was taken, as nearly as possible normal to the course of the river, and on that range a number of nearly equidistant stations were established, from 13 to 17 in number, from one bank of the river to the other. No. 1 was next the left bank. Observations were taken at these stations, in a more or less intermittent manner, from December, 1881, to November, 1882.

Plottings are shown in Figs. 6, 7, 8 and 9 of the velocity curves obtained by taking the means of the observed velocities at each station

during the year, at each of the observation points enumerated above, and also of the means of all the observations at each place. These results show a striking similarity in one respect, namely, in the resemblance between the curves of the different stations at the same observation point among themselves. At Columbus, for instance, the typical

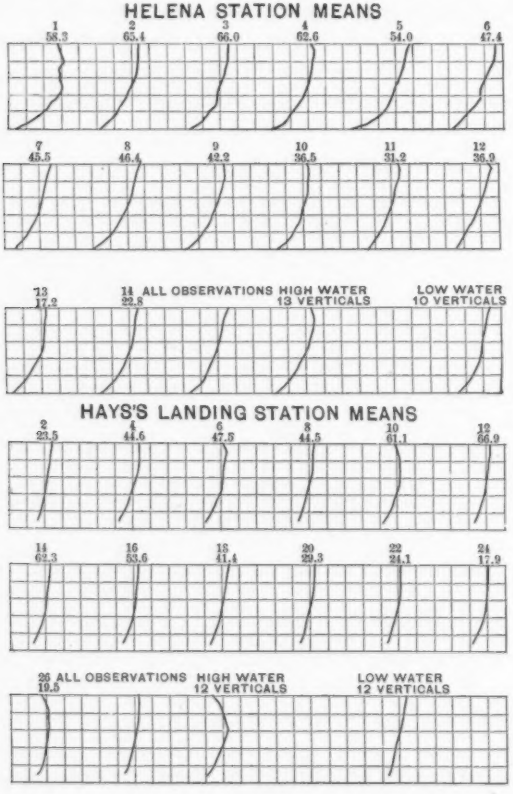


FIG. 8.

form of curve displayed by the combination of all the observations is distinctly traceable at each of the stations. At Paducah there is a little more diversity, owing to the peculiarity of the situation just below the mouth of a great tributary. Generally, the family likeness between the

station curves is very strong, and this notwithstanding differences in the depth of the water. At Helena, for example, the curve at Station 2, where the average depth throughout the season was 65 ft., is of the same type as at Station 13, with an average depth of 17 ft. So Hays's Landing, Station 12, with a depth of 67 ft., presents a curve which does not differ essentially from that shown by Station 24, with 18 ft. of

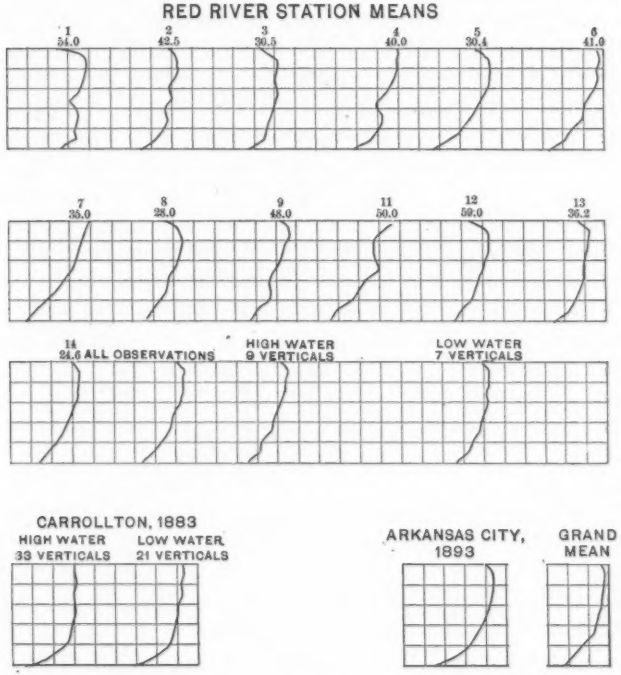


FIG. 9.

depth. One important difference there is, namely, that the minimum or bottom velocity often bears a closer ratio to the maximum in small depths than in great, a point to which attention may be directed hereafter. At Red River Landing the observations were so few (usually one or two at each station) that the means are very irregular, and are hardly more than individual observations. Fig. 9 also shows plottings

of Mr. Price's curve at Carrollton in 1883, and of Mr. Baily's at Arkansas City in 1893.

Ratio Between Mean and Mid-Depth Velocity.—The results of the observations of 1882, as far as they go, do not confirm the conclusions of Humphreys and Abbot with regard to the constancy of the ratio between mean and mid-depth velocity, nor as to the numerical value of that ratio. There are no mean velocities in the individual vertical planes reported. The ratio between the mean of the mean velocities in all the verticals at each observation point and the mean of the mid-depth velocities in the same verticals varied from 0.937 to 0.981. The mean of all observations is 0.958. At Paducah, Mr. Twining gives the mean coefficient for the reduction of mid-depth to mean velocity, in vertical planes, as 0.965, varying, however, for the several stations, from 0.944 to 1.000, and for the two sides of the river from 0.955 to 0.975. At Carrollton, in 1883, Mr. Price found the average percentage of mean velocity to mid-depth velocity to be 0.9625, being a little less for a rising river and a little more for a falling river. The range of values, for means, was from 0.9650 to 0.9571. Mr. Powless's observations at Fulton in 1879-80 were taken with imperfect apparatus (double floats), and seem hardly comparable to those of later date. They give a ratio of nearly unity, the mean velocity being sometimes the greater. The factor used at Arkansas City for reducing mid-depth to mean velocity is 0.9470. At Wilson's Point it is 0.9550. At Red River Landing the ratio 0.9572 is used.

The method suggested by Humphreys and Abbot for abridging the operation of taking discharges was to take velocities at mid-depth only, and to reduce the quantity thus obtained to mean, in each vertical, by the use of the coefficients found by them. As the relation, as announced by them, was so close, in the early observations under the Mississippi River Commission, namely, prior to 1883, the mid-depth velocity was assumed equal to the mean, and the discharges of 1879-80 and some of those of 1882 are computed on that assumption. In 1882, however, the velocities seem to have been obtained, in great measure, by the integration method. The reduction and discussion of the observations in vertical planes made in 1882, however, showed that this assumption was far from correct. At Paducah the mean velocity was found at 0.63 of the total depth, at Columbus at 0.71, at Helena at 0.61, at Hays's Landing at 0.60, at Red River Landing at 0.61. The

mean of all was 0.63. In Mr. Price's Carrollton observations of 1883, the mid-depth velocity was diminished by 3.5 to 4%, according to circumstances. In 1884-85 and subsequently, up to a recent date, the velocities were taken at 0.6 depth, and the results accepted as the mean velocities for those verticals without correction. At the present day a correction is sometimes applied, though not uniformly.

The method of ascertaining the mean velocities in verticals in 1882, whereby to compare them with velocities at different depths, was usually by integration, as practiced by Mr. Révy in 1871,* and, perhaps, by others; that is, the meter was lowered from the surface to the bottom and then raised from the bottom to the surface, the total number of revolutions being counted and the time elapsed being noted. The total number of revolutions divided by the time gives the mean velocity in revolutions in that vertical section.

Velocities Taken at Different Depths.—There are some objections to taking velocities at so great a distance from the surface as 0.6, particularly at high water, when depths are very great and currents very strong and turbulent, the meter under very imperfect control, and its behavior entirely exempt from scrutiny. Since a coefficient of reduction has to be applied anyhow, the innovation has lately been introduced of taking observations at various depths as might be most convenient, or at a fixed depth of 20 or 25 ft., as in flanking, and reducing by a factor. This method was used in 1893 at Arkansas City and Wilson's Point, and the results seem to have been more accordant and more satisfactory than usual. The factors of reduction used are given for the information of the profession. At Arkansas City: At 0.1 depth, 0.8978; at 0.2, 0.8966; at 0.3, 0.9081; at 0.4, 0.9272; at 0.5, 0.9470; at 0.6, 0.9831; at 0.7, 1.0411; at 0.8, 1.0964; at 0.9, 1.2339. At Wilson's Point: At 0.1., 0.9136; at 0.2, 0.9154; at 0.3, 0.9269; at 0.4, 0.9349; at 0.5, 0.9550; at 0.6, 0.9897; at 0.7, 1.0295; at 0.8, 1.0930; at 0.9, 1.1959. It would undoubtedly conduce to accuracy to use separate factors for great and small depths, from the fact that the curves of vertical velocities often vary much with the stage of the water.

Imperfections of Method.—The imperfections of the ordinary method are principally these: It is assumed that the 0.6 velocity, corrected or uncorrected, represents the mean velocity in any vertical plane, and it is assumed that the velocity varies uniformly from station to station in

* See his "Hydraulics of Great Rivers," pp. 17, 18, etc.

a horizontal plane. In the grand mean it is probable that these assumptions are nearly correct. In any individual case they are likely to depart widely from the truth. At Columbus, in 1882, the mean velocity was found at a depth of 0.71. At Carrollton, in 1883, at high water, the place of mean velocity was at a depth of 0.77; at low water, 0.66. It cannot be safely assumed, then, that the place of mean velocity is to be found at the same depth at all stations or at the same station at different stages; nor does there seem to be any good reason why such a result should be expected. The curves of vertical velocity, as plotted from the individual observations of 1882, differ widely from one another. Some show the mean velocity at 0.3 depth, some at 0.8. The error introduced by assuming 0.6 is sometimes as much as 0.5 ft. per second. It is usually much less than this, not more than 0.1 ft. The error consequent on the assumption of uniform variation of velocity in a horizontal plane will, of course, depend on the number of observation points. Some experiments were made by Captain Townsend, in 1893, with a view of determining the probable error caused by taking stations too far apart. Discharges were accordingly calculated with the stations 150 ft. apart, and afterwards with the intermediate stations omitted, making the points where the velocity was measured 300 ft. apart. The results are given on page 2949 of the "Report of the Mississippi River Commission for 1894." The greatest difference found was about 3 per cent. Ordinarily it was less than 1 per cent. The excess was not uniformly on either side.

It is evident that in any individual case it is a violent assumption that a single observation, at no matter what depth, will represent accurately the mean velocity of a partial area of 2 000 to 20 000 sq. ft., and that 20 such observations will give the mean velocity of a turbulent and irregular river of a cross-section of 200 000 or 250 000 sq. ft., and a discharge of 1 500 000 cu. ft. The errors may compensate one another, or they may be cumulative. In short, it is clear that anything like minute accuracy in the measurement of the actual discharge is out of the question by the means heretofore employed. It is also clear that very gross discrepancies in individual discharges should not cause surprise.

It is very possible, however, that the results attained by present methods are more accordant than they would have been if the actual discharge had been correctly ascertained; for they partake of the

nature of mean results, and are thus approximated somewhat to one another. From what has been already learned, it will appear that the actual discharge itself varies considerably from day to day and from moment to moment. It is influenced by the supply from tributaries and the return flow from basins, which is of itself very irregular, depending on accidents of rainfall, etc., by winds, by disturbances of cross-section due to caving and the passage of sand waves, by changes in the conformation of the shores, by the shifting of currents due to the change from low-water to high-water channels and the reverse, and doubtless to many other causes not understood or suspected. Ebbs and flows are frequently observed in the reading of gauges. Even on still days the gauge will fluctuate a tenth or so while the observer is looking at it. The existence of what have been called pulsations of the current has been affirmed and denied. In the case of winds, it is known that by their influence the thread of the current is shifted both vertically and laterally. In fact, every record gives evidence of what are called great irregularities in the motion of the water. Several instances of this kind have already been cited. It is true that these correct themselves in time, the slope being the great regulator; but this cannot prevent oscillations; it can only keep them from becoming excessive. As an instance of extreme departure from mean conditions, on May 3d, 1892, at Arkansas City, the measured discharge was 1 532 875. Next morning it was found to be 1 733 676. Astonished at this enormous discrepancy, the engineers repeated the observation on the afternoon of the same day, and found 1 741 526. The next day, May 5th, it was found to be 1 589 172. At Wilson's Point, 93 miles below, there is no trace of such a perturbation on that or the several following days. It was therefore momentary or altogether illusory. In 1893, at Arkansas City, an excessive discharge, as found by the ordinary method, was corrected by the flanking method.

Results.

The thirteen published volumes of the "Reports of the Mississippi River Commission" contain a vast mass of heterogeneous data on the subject of discharges, carefully reduced and tabulated in a suitable form for use. They cover pretty nearly every stage and every condition, and constitute a great storehouse of information. On attempting to make use, however, of these laboriously acquired facts, the investigator is

met at the very outset with a grave doubt whether they are facts or not. Many of them, as will be found on the most superficial examination, are flatly contradictory of one another. Therefore they cannot all be true. This being the case, which are true and which are false?

It is found that a discharge of 1 400 000 passed one place and of 1 200 000 passed another at only a day's interval, for two weeks together; that a discharge which passed last year at a gauge reading of 41 passes this year at 35; even the different seasons of the same year will witness an almost equal disparity. A conclusion drawn from the careful study of the observations at one place is completely upset by the teachings of another. The coefficient of the formula varies from 145 to 95 at the same place and stage, and within two weeks. The discharge curve is like a skein of tangled thread.

The first feeling of the investigator who plunges into this mass of perplexity is one of profound skepticism. There is no truth in it; or if there be any, it is impossible to distinguish it. Gradually, however, he finds that certain features are constantly reproduced; that even in the most complicated forms there are shadowy outlines, like those of composite photographs. Even the most chaotic-looking curve has a general direction, a tendency toward a central line or axis. He finds that what he rashly pronounced to be lies are often profound truths, concealed or deeply overlaid, it may be, by films of error, or distorted by temporary perturbations. What he wants is there, if he can only get at it.

Of the long list of discharges given in Table No. II, two series are particularly valuable; those of 1882, from their great extent, both in range of river and in time, and from the fact that they contain the records of the greatest known flood; and those of 1884–85, because they also cover a wide range from high to low water, and because they have been thoroughly discussed, and there is a pretty accurate knowledge of the methods practiced and the probable sources of error in them. Next to these in importance are the high-water observations of late years, in which it is possible to study the effect of the closer confinement of the flood waters and the greater heights thereby attained. The observations of 1888–89, while extending over several months, embrace only a limited range, being confined to perplexing and indecisive medium stages.

Objects Sought.

The immediate object is to find out, first, what was the greatest actual discharge that ever passed down the Mississippi, not only that which was held in the channel, but also that which passed in the swamps and sloughs outside of the banks; and, second, what height would such a flood have reached on the gauges of the several stations if it had all been restrained between banks or levees. Better than all would it be to find out the general law regulating the relations of the discharge to the gauge, so that it would be possible, not only to ascertain the height corresponding to the maximum discharge, but to any discharge, at any time, past, present or future. This last quest is the philosopher's stone of the river engineer.

The Flood of 1882.—In the year 1882 occurred what is considered the greatest flood that has ever been known, and if the discharge of that year had been measured with accuracy, half of the question would have been answered. But only a part of the discharge was measured, and that pretty crudely. In that year, the levee system was in a very imperfect, almost rudimentary, state. The Saint Francis Basin was not leveed at all. The White River Basin had only a few fragments of old and private systems. The upper portion of the Yazoo Basin had the remains of an old system in a ruinous condition. The lower part of the same territory had a continuous line, but weak and insufficient. The upper Tensas District (from Cypress Creek to a few miles below the boundary between Arkansas and Louisiana) had a broken and fragmentary line, as good as none. The Louisiana levees were small, weak and discontinuous. Consequently, long before the river had reached its height, it began to overflow its banks and to escape through gaps in the old embankments and frequently over their tops. As it rose still higher, it began to break the levees, and, finally, on the well remembered night of February 28th, a violent storm arose, which demolished the embankments of the lower Yazoo District in 30 or 40 places, and those of Louisiana in perhaps as many more. This was eight or ten days before the flood would have attained its culmination.

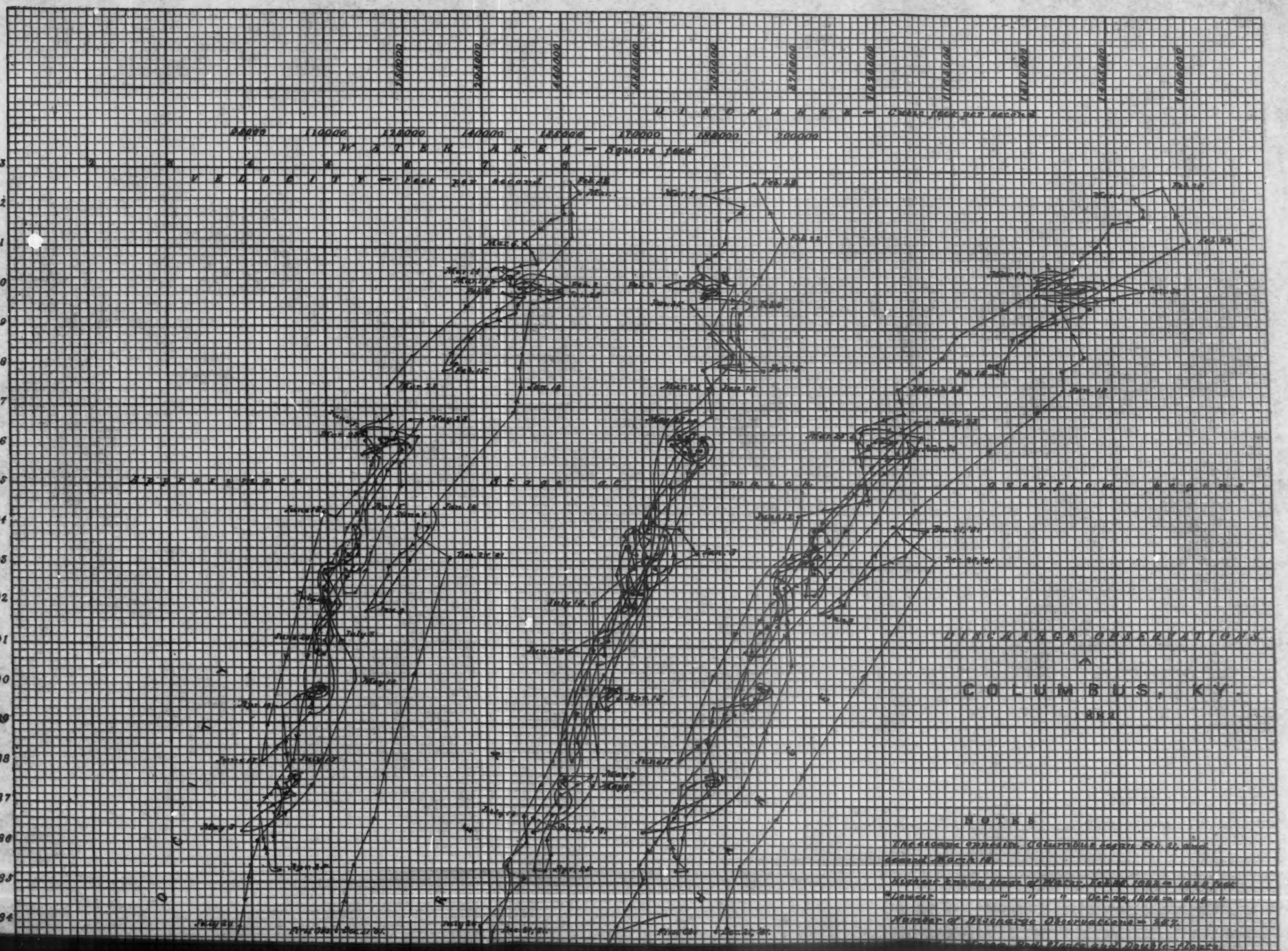
Of the vast volumes of water which escaped from the main river, large quantities found their way back to the channel. Of that which was abstracted immediately below Cairo, a considerable part returned in the bends immediately below, and was measured at Columbus.

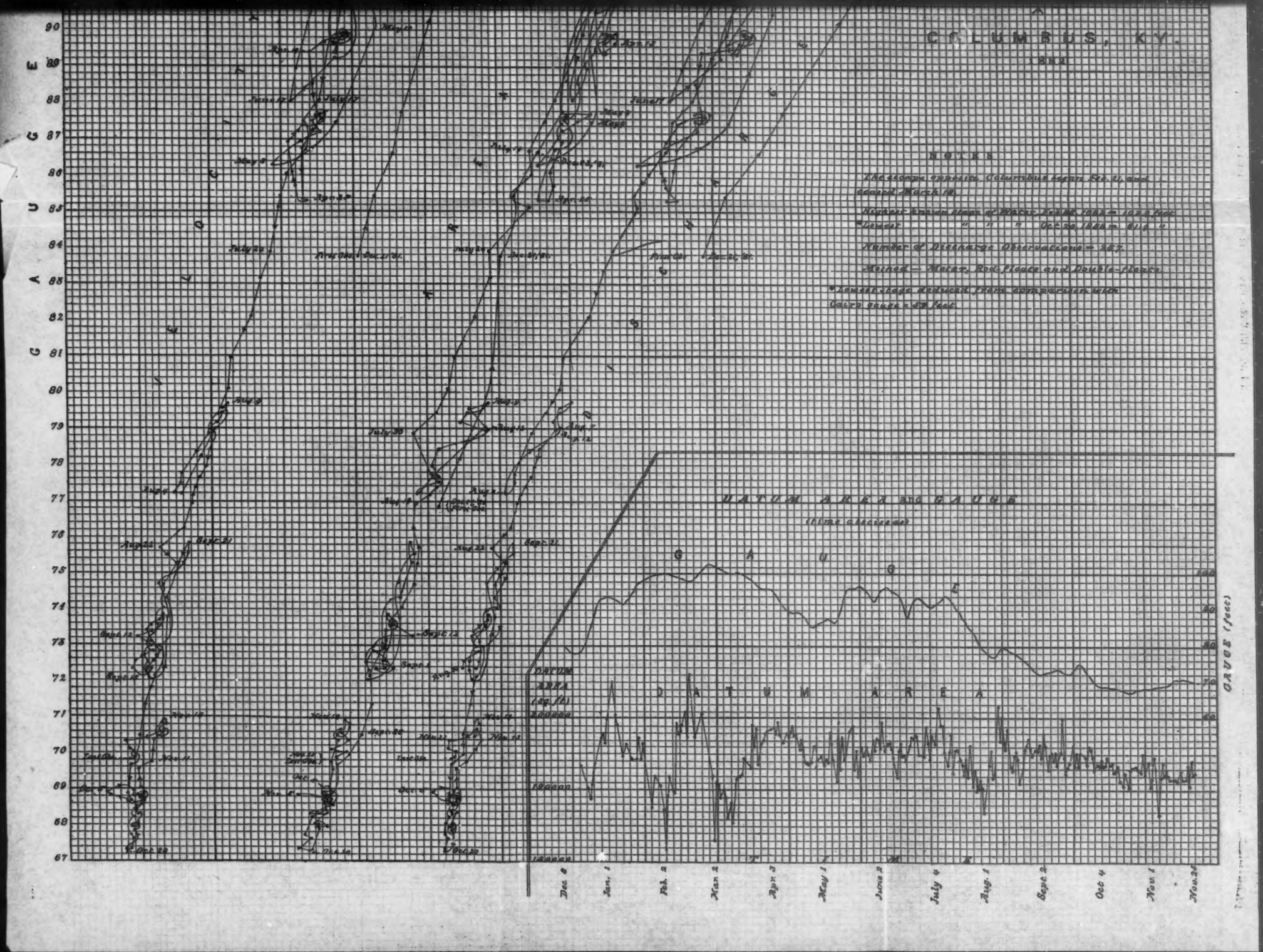
Nearly all came back into the river at New Madrid, but no discharges were taken there. Of that which poured into the Saint Francis Basin, all returned at or above Helena, except what was lost by evaporation or leakage or was retained in the lakes and swamps. That which escaped into the Yazoo Basin was returned, in the same manner, above Vicksburg. Of that which filled the White River Basin part was restored through the White and Arkansas Rivers, or at the foot of the basin above Arkansas City; part went to swell the enormous volume discharged into the Tensas Basin, to be partially returned through Red River, but mostly never to see the Mississippi again, being conveyed by the Atchafalaya and the low lands of the basin of the latter directly to the Gulf of Mexico.

Attempts were made to measure the escaping water as well as that which was retained in the channel, and in one way or another all of it was accounted for. It is very difficult to estimate the degree of credence to be attached to the crevasse and over-bank discharges. High-water discharges are attended with many sources of error, some of which have been indicated. Not only are the estimates of the volume of water passing liable to be deceptive, but the disturbances produced by crevasses, escapes and return flows have a very great effect upon the stage. Admitting, for instance, that 1 500 000 ft. of discharge passed Helena, yet it is probable that in a closely confined river it would have passed at a stage of 50 ft. instead of at 47. The behavior of a river while within its banks is comparatively regular. The discharges at such stages have been observed many times and at many places, and may be checked by one another or means taken. Observations made at such stages therefore afford the best means of studying the laws regulating the discharge, and the results arrived at by such reasoning may be applied, with proper modifications, to the case of a confined river at higher stages.

Discharge Curves.

To arrive at a preliminary conception of the law governing the progression of any series of data, a customary and excellent method is to plot those data to some argument that seems to have a connection with them and is itself known or easily ascertainable. This is an especially well recognized mode of procedure as regards discharges and velocities. Text-books say that discharge curves may be thus


DISCHARGE OBSERVATIONS
AT
COLUMBUS, KY.
1912
NOTES


COLUMBUS, KY.
1884
NOTES
The gauge opposite Columbus began Feb. 15, 1884 and ceased March 18, 1884.
Highest known stage of Water, Jan. 31, 1884 = 101.8 Feet.
Lowest " " " Oct. 20, 1864 = 61.6 Feet.
Number of Discharge Observations = 267.
Method = Meter, Rod, Float and Double-Float.
*Lowest stage determined from comparison with Cairo Gauge = 58 Feet.
DATUM AREA AND GAUGE
(Time Abscissae)
DATUM AREA (Sq. Ft.)
GAUGE (Feet)
Dec 8
Jan. 1
Feb. 2
Mar. 2
Apr. 3
May 1
June 2
July 4
Aug. 1
Sept. 2
Oct 4
Nov 1
Nov. 25

constructed in which each successive gauge height shall correspond to so many cubic feet of discharge.

These curves, says Debauve,* are parabolas, convex to the axis of ordinates (the gauge heights). The curve of mean velocities is concave toward the same axis. These, he continues, are precious indications as to the general form of curves of discharge and velocity, and they enable these curves to be traced with great accuracy as soon as two or three points have been determined carefully, and the correctness of a series of experiments to be checked, and anomalies and errors to be detected.

The Discharge Curves of the Mississippi Very Complicated.—Debauve's remarks are in the main judicious; and doubtless in smaller rivers, and especially those of fixed regimen, it is comparatively easy to trace such a curve. In the case of the Mississippi, such a process is attended with many difficulties. Not only are the data very discrepant, but, even with perfectly correct numerical estimates, the relations between gauge and discharge vary extremely from time to time, owing to the instability of the bed. There are always, in all rivers, variations in these relations due to the rising and falling stages; but with the Mississippi this is but the beginning of complications. As an illustration, let there be plotted the curve of 1882 at one of the stations, say Columbus. The result is shown on Plate VI.† The figure has a general trend, but as to attempting to fit any particular curve to it, or deducing from it any law, it is baffling in the extreme.

These bizarre-looking figures may be greatly simplified by selecting for plotting only such observations as were made at stationary stages, and thus eliminating the effects of the rising and falling movements. The adoption of this course will give an assemblage of points, through which a curve may be drawn that will fit them fairly well. Indeed, half a dozen such curves may be drawn, none of which will fit perfectly, but all tolerably, and all about equally well; but when an attempt is made to prolong them, to compare the relations which would exist at high stages, it is found that the upper branches will depart widely one from another.

The curious and complicated sinuosities and entanglements of the actual velocity curves are not inexplicable. Part of them are due to

* "Navigation Fluviale et Maritime," p. 53.

† From the "Report of the Mississippi River Commission for 1883."

errors of observation. All of them are more or less affected by such errors; but the greater part, by their persistency and evident connection, show that they are genuine. There is no use in attempting to average them away. On the contrary, they are worthy of careful study, and an investigation of them cannot fail to be profitable. For this purpose, and in order to be able to discriminate between errors and unexpected or unexplained truths, it will be well to go back a little to first principles, and this the more that there is not an axiom of hydraulics which has not been called in question at the present day.

First Principles.

Mean Velocity.—The mean velocity of a stream is an artificial quantity, not actually existing in nature. It is simply the quotient obtained by dividing the discharge by the area of the cross-section.

The fact that the mean velocity is an abstraction is often lost sight of. Such expressions as "the water prism passing such a point" are often heard, the water prism being supposed to be a mass of water having a base equal to the area of the cross-section at that point and a length equal to the mean velocity for the time considered. In reality no such water prism passes. There does, indeed, a quantity of water pass equal in volume to such a prism, but it passes in fillets, each of which has a different velocity from that which adjoins it. The upper layers of each vertical section pass at a speed which is frequently double that of the bottom layers, and the surface velocity in the main channel is sometimes four times that next the shore. Grave errors have often been committed, are committed every day, by inattention to these facts, palpable as they are.

Slope of the Water Surface.—If there is any one principle in hydraulics that one would think firmly established, it is that in open channels the motion of each particle of water depends on the slope of the water surface. Yet it has been disputed, not so many years ago, by an engineer of reputation,* who thought that the observed phenomena of the flow of rivers could not be explained by it. He accordingly returned to the doctrine of the older Italian hydraulicians, that velocity in a running stream depended on depth below the surface. Indeed, these

*Robert Gordon, M. Am. Soc. C. E. See his "Theory of the Flow of Water in Open Channels." Rangoon, 1875.

phenomena are so puzzling that one may be excused sometimes for a little heterodoxy.

The postulate on which all formulas for velocity rest is this: In uniform motion the accelerating forces are equal to the resistances. The application of these formulas to living streams depends on the greater or less approximation of the actual circumstances to those demanded by the hypothesis of uniform motion.

Permanent and Uniform Flow.—The essence of permanent motion of water is equality of discharge at the various cross-sections. Uniform flow is defined to be that particular case of permanent flow in which the sections are equal and the bed of the channel parallel to the water surface. If it be conceded that in a flowing stream, after a greater or less time, a condition is attained similar to that demanded by the postulate, then an equation may be found which shall express the equilibrium between the accelerating and the retarding forces. The accelerating force is gravity. The retarding force is supposed to be friction, either of the flowing liquid on the solid walls between which it runs, or of the particles of liquid among themselves, the latter constituting what is commonly called viscosity or viscidity.

Considering only the former element, it is thought probable *a priori*, and it has been approximately found by experiment that the resisting power of friction of solids on liquids varies directly as the square of the velocity.

Laws of Fluid Friction.—The mass of evidence on which rest the so-called laws of fluid friction is very small—incredibly small considering the importance of the subject and the momentous conclusions which have been drawn from the said laws; and their correctness cannot be considered firmly established. The laws are as follows:*

The frictional resistance is independent of the pressure between the fluid and the solid against which it flows.

The frictional resistance of large surfaces is proportional to the area of the surface.

At low velocities of not more than 1 in. per second the frictional resistance increases directly as the relative velocity of the fluid and the surface against which it moves. At velocities of 0.5 ft. per second and greater the frictional resistance is more nearly proportional to the square of the relative velocity.

* "Encyclopedia Britannica," Vol. XII, p. 482, article "Hydromechanics."

Formula.—From these laws and the laws which govern bodies falling under the influence of gravity it is easy to deduce a formula depending on slope, cross-section and wetted perimeter, or, in short, slope and mean depth. The result is the well-known expression,

$$v = c\sqrt{r\,s},$$

which embraces in itself pretty nearly all the hydraulic formulas which have been attempted to be devised for rivers and for ordinary velocities. In the Chézy formula c is supposed to be a constant. In most of the others it is a function of r or of s, or of some quantity not included in the formula, as the roughness of the channel, etc. It must be remembered that it refers strictly to uniform motion, and that if it be attempted to apply it to any other case it must be modified to meet its new requirements.

General Formulas.

There are several reasons which militate against the employment of this formula, in any of its modifications, for general purposes; that is, as applied to all sorts of rivers, all stages, all slopes, all forms of section. The objections apply, also, to the use of the formula for different stations on the same river; and some of them are adverse to the possibility of devising, for the Mississippi and rivers of similar type, a rational expression for the relations between the several variables, even for different stages at the same place. This does not exclude the hope of finding empirical expressions which may be useful for local and temporary purposes.

Objections to General Formulas.—First.—The hypothesis of uniform motion is not even approximately realized.

Second.—It is asserted and taken for granted that the stream is always in a state of equilibrium between the resisting and accelerating forces.

Third.—The formula is founded on the supposition that the only resistance encountered is that of fluid friction, which is subject to the laws cited above.

Fourth.—It is assumed that the symbol v represents the mean velocity.

Fifth.—It is assumed that the bed of the stream, even though irregular, is unalterable. It will be seen that this is very far from true,

and that the changes in the area of the cross-section have a serious disturbing effect on the velocity.

Sixth.—For practical purposes, the only expression for the slope that is available is the slope taken over long distances, such as is estimated, for instance, from one gauge station to another. In the application of a formula, it must be assumed that this slope is the same as the local slope, *s*, of the formula, or bears a definite relation to it, which is not correct, unless by accident. Even the local slope is not constant for all parts of the cross-section, but varies greatly from bank to bank.

It will appear more clearly how very far the actual condition of the river departs from uniform motion. It is said, however, that the hypothesis may be applied on the supposition that the portion of the stream considered is so short, unobstructed and regular that it shall conform nearly to the required condition. This would confine the formula, strictly speaking, to one stretch at a time ; or, at most, its use would be limited to localities which were exactly similar.

Equilibrium.—It is usually asserted and taken for granted that rivers are in a state of equilibrium, and this is the foundation of the theory of uniform motion. If a stream of water flow down an inclined plane, the velocity is accelerated by the action of gravity at a uniform rate. The resistance of friction, on the other hand, increases as the square of the velocity. Consequently, as the velocity increases, the resistance increases more rapidly, and a time comes when the two are equal ; and after that the motion is uniform, the work of gravity being expended in overcoming the friction of the bed.

Dubuat's Experiments.—It is well known that most of the experiments on which is founded the present knowledge of the motion of water were made on a very small scale. The greater part of Dubuat's famous series of observations were made with troughs less than 20 ins. wide, with a depth of water not exceeding 1 ft. It is wonderful what solid, valuable and even brilliant results were achieved with such imperfect means. It is almost equally wonderful how many inferences hastily drawn from such imperfect data (not, however, by Dubuat himself) have been accepted as facts. It is no doubt true that, given a channel of infinite length and unerodable and unchangeable bed, no matter what the cross-section or what the inclination, at some time a state of equilibrium would be reached. The smaller the stream, the

sooner would this condition occur. With very small channels it may take place within a few feet. In Bossut's experiments, made with a wooden trough 656 ft. long, even with so great a slope as 1 in 10, equilibrium was attained within the first 108 ft.* In very great rivers it may not occur at all. In fact, it is probable that it does not occur. If it did, then no matter what the relations between the resistance and the velocity, the equilibrium, being established upon a certain slope, would continue to subsist upon the same slope.

In fact, the equation $R = W \sin. i$, which expresses the equality between the resistance and the effective component of the weight, or the accelerating force, is impossible except in strictly uniform motion; that is, when the cross-sections are not only equivalent in area, but are similar in form, with the water surface parallel to the river-bed. The equation merely expresses the proposition that in a stream of uniform bed, section and slope, in which a condition of equilibrium has been attained, the velocity will be expressed by the relation $v = c\sqrt{rs}$. If any one of the conditions fail, the equation ought not to, and will not, hold good. If it be true of one shape of cross-section, it will not be true of another shape; if it be true of high-water conditions, it will not necessarily be true of low-water conditions; that is, in these cases c will not be a constant, or will not be the same constant at Helena as at Carrollton, at high water as at low. If it be true of a stream in which equilibrium has been reached, it will not be true when that condition has never existed or has been disturbed.

Modified Equilibrium.—Though the ideal equilibrium of the formula does not exist (for that would imply uniformity of slope), yet that there is a quasi-equilibrium is evident from the fact that the high-water velocity is nearly the same, in the mean, from the head to the foot of the alluvial valley; in other words, the accelerating and retarding forces balance one another in the long run. The accelerating force is always gravity; but the effective component of that force varies widely with the perpetual changes of local slope. The retarding force, too, is always of the same nature. It is the resistance of the bed; but this, in a natural stream, is very different from the supposed friction of a uniform and regular section in a straight reach with a fixed inclination. It is rather composed of a series of shocks or paroxysms, each followed by a recovery and a subsequent accelera-

* D'Aubuisson, "Hydraulics," p. 116. (American edition, by Bennett.)

tion. The plunge down a steep slope, *A B* (Fig. 10), is followed by a shock against the bottom at the change to a flatter inclination. The swift impulse in the nearly straight part of the bend *A B* is followed by the shock against the bank in the bight of the bend *B C*. Each of these shocks brings the velocity to a check, which is immediately succeeded by an acceleration where there is a renewal of the conditions favorable to the same. There is a series of oscillations. Each acceleration speedily brings about its own correction, which is rather an over-correction; and each retardment produces the conditions of a subsequent acceleration.

In a stream thus organized, a system of general equilibrium may be said to be established, but it must always be borne in mind that the expression is true only in a limited and qualified sense, and that the equilibrium is liable to disturbance whenever there is a change of circumstances.

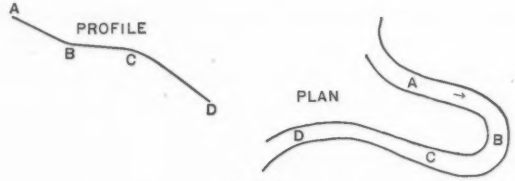


FIG. 10.

The formation of the bed of a sedimentary stream depends on the combined influences of all the different stages. Of these, the high-water stages are by far the most powerful, from both volume and velocity. The living force at extreme flood is more than a hundred times what it is at extreme low water. At bank-full stage it is twenty times as much as at ordinary low water, and five times as much as at medium stages. If it is assumed that in every 10 years there occur two very high waters, five reaching bank-full stage, ten in which mid-stage prevails and ten of ordinary low water, and if extreme high water be supposed to occupy one month, bank-full stage two months, mid-stage four months, and low water three months, the respective formative forces of the several stages for the 10 years will be about as 12, 50, 20 and 4; the great forces which prevail in extreme flood being partly compensated by the rarity and the short duration of the latter; and the frequency and length of ordinary floods and medium stages giving them an influence to which their dynamic importance would not

entitle them. On the whole, it is clear that the stages greater than the medium control the general features of the bed, and that the lower stages subject it to certain modifications which, in turn, are removed at the next flood.

There is no necessary antagonism, in general, between the high-water and the low-water forces. The general slope and general cross-section of the stream are not affected by the transition. It is not clear that any general fill or general scour is to be attributed to either stage. The modifications which occur from changes of stage appear to be local. They consist mostly in the building of the bars and the deepening of the pools in time of flood and the reversal of these processes in low water.

Changes of Water Slope with Stage.—While, by these operations, the general dimensions and slope of the bed are not affected, unless by an unusual prolongation of the high or low stage, or an unusual succession of high-water or of low-water years, yet the local modifications of them are considerable; and more than this, the water slope, which in flood exhibited an approximation to uniformity, and a certain independence of minor inequalities of the bed, in low water conforms more strictly to those inequalities, and great variations of local slope are introduced. The general equilibrium is not disturbed, or rather, there is substituted for it a different sort of equilibrium. There is no longer the violent shock against the banks or bottom. It is not necessary, and the conditions are not in force to produce it. The equilibrium which now exists approximates more nearly to the ideal equilibrium of uniform motion, with the necessary modifications due to violent changes of cross-section and slope.

If Equilibrium Exists at High Stages, It Will Exist at Low.—Let there be given a certain channel of uniform dimensions and of fixed slope, and let it be conceded that after a certain point has been reached a stream as full as the channel will bear will be in a state of equilibrium, the resistances balancing the acceleration. Let now the stream be reduced in size; it will still be in equilibrium. No matter how low the inclination of the channel nor how small the stream, if the greater volume has been passed, the less will be. But the resistance of friction will be disproportionately great, the velocity disproportionately low, and the area of the cross-section consequently increased. It will take a shorter time for the small and low stream to reach equilibrium

than for the large and rapid one. According to Dubuat's experiments, when equilibrium was established, the velocity then attained was equal to that due to the entire fall, and the head due to the mean velocity was equal to the difference between the height of the reservoir whence the canal was supplied and that of the uniform section.* The bottom of the reservoir and that of the canal were at the same level.

If, therefore, a channel be of such dimensions and slope as to carry a large stream in equilibrium, it will carry a small one. The resistance due to the area of the rubbing surface will be greater in proportion to the volume carried in the latter case than in the former, but the resistance due to velocity will be less. If the wetted perimeter be reduced from 4 000 to 2 000 and the volume from 1 500 000 to 100 000, then the proportion of rubbing surface to volume will be increased 7.5 times. If the velocity be reduced from 6 to 2.2 ft. per second, then the resistance due to velocity will be reduced 7.5 times. In short, the proportions will be so adjusted that a new equilibrium will be established.

Experiments on a Small Scale Not Necessarily Applicable to Great Rivers.—The conclusions drawn from experiments on a small scale cannot be applied, except by analogy, to great rivers, or perhaps even to small ones; and in applying the analogy, due heed must be paid to all circumstances. It cannot be inferred, for instance, in the case of the Mississippi, supposing the origin of velocities to be Cairo, that an equilibrium would be attained as soon as a mean velocity of 6 ft. was reached, the fall due to which is 0.56 ft., corresponding to about a mile and a half of distance. If such were the case, the slope, as has been intimated before, would be constant below this point. But would not a curved slope carry the discharge as well? Undoubtedly; but in the case of equilibrium, the velocity, under the circumstances of a decreasing slope, would be reduced and the cross-section increased as the slope became flatter and flatter, which is known to be the reverse of the truth, and which would, indeed, be incompatible with the very existence of a silt-bearing stream, as it would not permit the transportation of its load of sediment. Such a condition would lead to the gradual and pretty rapid filling up of the lower reaches of the river.

The means by which the velocity is maintained nearly constant under the continual changes of slope are several. In that part of

* D'Aubuisson, "Hydraulics," p. 136.

the river where the slope is steepest, the cross-section is wide and shallow. Bars and islands are plentiful. Large volumes of water are lost over "points" in flood time, or escape into the great basins and are returned at the outfalls of the same. There are violent alternations of width, deep bends, frequently with banks of inerodable or difficultly erodable material. As the slope becomes flatter, the cross-section becomes more favorable to the passage of the discharge, deeper, narrower, until, as the Gulf is approached, it has not much more than half the width that it possessed above Helena, and has almost double the depth. It is also somewhat straighter and has much easier curves and a more symmetrical and more uniform cross-section.

The resistances above enumerated are very difficult to be reduced to a formula. While it may be said that the sum of them is equal to the accelerating force, yet how is the sum of them to be obtained? Resistances of this kind cannot be classified under the head of friction. The assumption cannot be made of them that they are independent of pressure or weight. They partake, in fact of the nature of work.

Fluid Friction and Viscosity.—It has been seen that the resistances to the flow of water are of two classes; one depending on the solid surface with which the water is in contact, and the other on the internal relations of the particles of the fluid among themselves, and these have been called by the names of friction and viscosity. If these terms are to be understood as embracing between them the total resistance, they must be accepted in very different significations from those which they usually bear. It is evident, from what has just been said, that pure friction is responsible only for a part and possibly a small part of the resistance of the walls of the channel; and it appears certain that pure viscosity cuts a very insignificant figure in the internal resistances.

If the motion of water took place with perfect regularity, if the sides of the channel were perfectly smooth and the layers or laminæ slid on one another without disturbance, the effect of viscosity would be comparatively slight. This fact was adverted to by Dupuit, in his discussion of the relations between bottom and mean velocity.*

Says Professor Unwin:†

"Boussinesq has shown that the central filament in a semi-circular canal of 1 m. radius, and inclined at a slope of only 0.0001, would have

* "Mouvement des Eaux," p. 55.

† "Encyclopedia Britannica," Vol. xii, pp. 459–460.

a velocity of 187 m. per second, the layer next the boundary remaining at rest. But before such a difference of velocity can arise, the motion of the fluid becomes much more complicated. Volumes of fluid are detached continually from the boundaries, and, revolving, form eddies traversing the fluid in all directions, and sliding with finite relative velocities against those surrounding them. These slidings develop resistances, incomparably greater than the viscous resistance due to movements varying continuously from point to point. The movements which produce the phenomena commonly ascribed to fluid friction must be regarded as rapidly or even suddenly varying from one point to another. The internal resistances to the motion of the fluid do not depend on the general velocities of translation at different points of the fluid (or what M. Boussinesq terms the mean local velocities), but rather on the intensity at each point of the eddying agitation. The problems of hydraulics are therefore much more complicated than problems in which a regular motion of the fluid is assumed, hindered by the viscosity of the fluid."

Indeed, many of these phenomena are not dependent on viscosity at all, for movements of an analogous nature are found even in air, such as eddies, whirls, and even the drawing-on of one stratum or layer by the friction or attraction of the one adjacent.

Dupuit's Objection.—It has been denied that the formula gives the quantity sought, for the reason that the v of the formula does not represent the mean velocity, but the actual velocity of that particular part of the stream that rubs against the bank. It is to the square of this velocity that the resistance is proportional, and it is this quantity that enters into the equation.*

The force of this objection seems to be somewhat diminished by the fact that in the experiments on which is based the knowledge of the laws of fluid friction, the velocity measured was not that actually existing at the surfaces of contact between the solid and the liquid bodies. Most of these experiments were made by moving solid surfaces through liquids, and the velocity which was proportional to the resistance was the velocity of the surface relatively to the banks, not relatively to the velocity of the envelope of water immediately adjacent to the solid, which participates in the movement of the solid. It may be fairly asserted that a part of the resistance experienced by the moving surface consists in imparting this motion to the surrounding water against the cohesion of the adjacent particles; and that conse-

* Dupuit, "Mouvement des Eaux," pp. 4-5.

quently the friction between the solid surface and the film in immediate proximity to it is not the friction sought, nor is the relative velocity of the two the velocity sought. In like manner, when the liquid is in motion and the solid stationary, the same reasoning may apply; that is, the bank of the river has already done a part of its work in retarding the velocity of the fillets immediately adjacent to it, and of those more remote fillets to which the influence of the adjacent fillets extends; and that hence the velocity of those retarded fillets should not be the criterion of the resistance experienced. If it were, then the velocity of the layer immediately next to the bank should be the one considered. Now this is nothing, or next to nothing.

Velocity of the Formula not the Mean Velocity.—There is undoubtedly, however, a flaw in the reasoning by which the velocity of the formula is made to appear to be the mean velocity of the whole section. The velocity and the resistance to which it is proportional vary at every point of the perimeter. At *A*, Fig. 11, the velocity is 1 ft.; at *B* it is 5 ft. If the perimeter be divided into n nearly equal spaces, then (supposing the proportion constant) the resistance in each part of the perimeter will be equal to $Al\frac{p}{n}v^2$, and the total resistance will be equal to $Alp\frac{(v_0^2+v_1^2+v_2^2+\ldots.v_n^2)}{n}$, where A represents the constant of proportion, l the length of the stretch of river considered, and p the wetted perimeter. Now the mean velocity is equal to $\frac{Bv_0+Cv_1+Dv_2+\ldots Nv_n}{B+C+D+\ldots.N}$, where B, C, D, etc., represent the partial areas. In the ordinary way of misunderstanding the formula, the total resistance is made equal to $Apl\left(\frac{Bv_0+Cv_1+\ldots.Nv_n}{B+C+\ldots.N}\right)^2$ instead of to the expression first given above. In the one, v^2 is taken to mean the average of the squares of all the partial or local velocities at every point of the perimeter; in the other it represents the square of the sum of the products of each individual velocity by its partial area, divided by the square of the total area. This error is not always of great consequence at high stages, but with particular forms of sections may

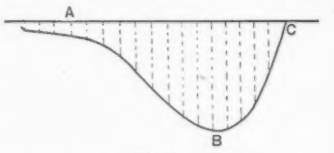


FIG. 11.

become very important. In one instance (Helena, in 1884–85), it amounted to 0.12; in another (Warfield's Point, 1893), to 0.14; but in still another (Bachelor's Bend, 1893), taken on the same day as the preceding, and only a few miles distant, to 1.31; all this at mid-stage, with a discharge of about 700 000.

It is well known that there is great difficulty in applying the Chézy formula or any formula to streams of different depths or even to the same stream at the same station at different stages of water. Observation shows that in such cases c is not a constant at all, but varies with the stage and with other circumstances. Will Dupuit's supposition afford a relief from this difficulty ?

Ratio Between Mean and Bottom Velocities.—The relation between the mean and the bottom velocities at different stages is not well understood. The ratio between the maximum, mean and minimum velocities has often been given in the text-books as about 5, 4 and 3, irrespective of depth. It was inferred by Dubuat, from his experiments, that the ratio of velocity at the surface to that at the bottom is greater according as the velocity is less, and that this ratio is entirely independent of the depth. He also observed that the mean velocity was a mean proportional between the surface and bottom velocities. This question has been made the subject of an elaborate investigation by Dupuit* who deduces the following formula:

$$\text{Bottom velocity} = \frac{\text{mean velocity}}{1 + 0.73\,r}.$$

If the ratio between the mean and bottom velocities be nearly constant at all stages, it makes little difference which signification be attached to the symbol v. The only difference between the use of the mean and the bottom velocity is that the coefficients will be less in the latter case, but they will differ among themselves by the same ratio.

Observations of 1882.—There are not many data from which to form a decisive opinion with reference to the relation between the mean and the bottom velocity at different depths. The observations of 1882 furnish a mass of information, some of which can be made to bear upon this point, or rather as to the relation between maximum and bottom velocity. By combining the different verticals, there will be found for this relation:

* "Mouvement des Eaux," p. 55.

At Paducah.

At depths	above 60 ft.....	5	observations,	0.47
"	between 50 and 60 ft....	5	"	0.49
"	" 40 and 50 "....	6	"	0.56
"	" 30 and 40 "....	21	"	0.56
"	" 20 and 30 "....	61	"	0.63
"	" 15 and 20 "....	27	"	0.67
"	" 10 and 15 "....	62	"	0.74
"	below 10 ft.....	24	"	0.76

At Columbus.

At depths	above 60 ft.....	55	observations,	0.65
"	between 50 and 60 ft....	33	"	0.64
"	below 50 ft.....	12	"	0.60

At Helena.

At depths	above 60 ft.....	19	observations,	0.58
"	between 50 and 60 ft....	23	"	0.53
"	" 40 and 50 "....	15	"	0.54
"	" 30 and 40 ". ..	13	"	0.58
"	" 20 and 30 "....	15	"	0.68
"	below 20 ft.....	5	"	0.71

At Red River Landing.

At depths	between 50 and 60 ft....	6	observations,	0.64
"	" 40 and 50 "....	4	"	0.66
"	" 30 and 40 "....	10	"	0.49
"	" 20 and 30 "....	8	"	0.66
"	below 20 ft.....	4	"	0.57

At Carrollton, in 1883, the mean of 33 high-water verticals gave a ratio of bottom to maximum velocity of 0.55. A mean of 21 low-water verticals gave a ratio of 0.49.

These results are not at all accordant, and there is not much profit in combining them further; especially as there is no direct method of comparing either the bottom or the maximum velocity with the mean. At Carrollton, Mr. Price found the ratio of mean to bottom velocity at high stages to be 0.58, and at low stages 0.53. However, Carrollton is

subject to very peculiar low-water conditions. For the whole series of observations, the ratio between maximum and bottom velocities is about 0.66, which corresponds very closely with that found by M. Bazin. Dupuit's formula gives, for depths of 60 ft., a ratio of mean to bottom velocity of 0.69; for a depth of 20 ft., 0.87.

This result M. Dupuit admits with great candor is not in accordance with experience, and he attributes the discrepancy to the internal movements of the particles above referred to.

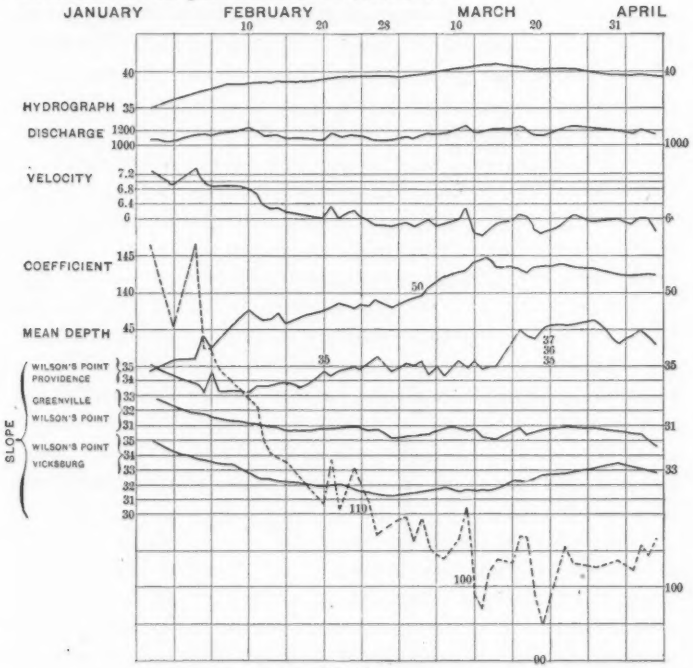


FIG. 12.

It cannot be affirmed, then, that the substitution of the bottom for the mean velocity would bring the formula into any closer accord with facts.

Variations of Cross-Section.—The variations in cross-section which progress during each season are accompanied by disturbances of the velocity which are not only unrecognized by the formula, but are apparently in direct opposition to it.

In any formula of velocity which has ever been proposed, if the slope remain constant (other circumstances being unchanged) and the mean depth be increased, the velocity will increase. Yet the observations of every year, at every point, show instances where increase of depth is accompanied by loss of velocity, not only where other circumstances seemed to be unchanged, but where they apparently conspired to aid the increased depth in bringing about an augmentation of the velocity. A remarkable example of this kind is given in Fig. 12, which is a plotting of the high-water discharge of 1890 at Wilson's Point. The phenomena are perfectly explicable, and, indeed, are perfectly natural, but they are not at all in accordance with the formula.

The theory of permanent motion demands as its first requisite uniformity of discharge. Indeed, it is impossible that streams could continue to flow, as experience proves they do flow, on any other supposition. The discharge that passes *A* must be, on the whole, the same that passes *B*, excluding, of course, disturbances from rises or falls. If it were otherwise, if at *A* there passed even 10 000 cu. ft. per second more than at *B* for 24 hours continuously, there would be an accumulation between *A* and *B* of 864 000 000 cu. ft.; and if these stations be 40 miles apart there would be a rise throughout the reach of 1 ft. Experience shows that the movement of the discharge takes place with remarkable uniformity. At stations which are contiguous, or which are even hundreds of miles apart, provided no tributary or outlet intervene, the gauges move from day to day with almost parallelism, subject only to the gradual oscillations of slope which accompany rises and falls.

Incompatibility of Conditions.—But if the discharge be uniform and the cross-section vary, the velocity must vary also, but in an inverse sense. The greater the area, the less the velocity. In the instances cited the increase of area was all in the direction of depth, the width remaining constant. Therefore, the condition of permanency of discharge seems to be diametrically opposed to the fundamental postulate demanded by the formula, the increase of velocity with mean depth.

Variations of Local Slope.—While the general slope does not vary materially during these operations, yet it is probable that the local slope experiences considerable changes. It will be seen in many instances that local slope is constantly working and is of great power. It is certain that the local slope corresponding to a high velocity can-

not be maintained when the velocity is considerably lessened. Information on this point is very scanty, and, indeed, the difficulty of defining what constitutes local slope and of ascertaining it after it has been defined, and of separating it from eddy action, etc., is very great; nor is it apparent that for the present purpose the benefit derived from such observations would be commensurate with the trouble involved in making them. In truth, at any given point, the influences of velocity of approach and of local slope are inextricably intermingled; and it is probable that the relative importance of the two elements is frequently interchanged at the same station.

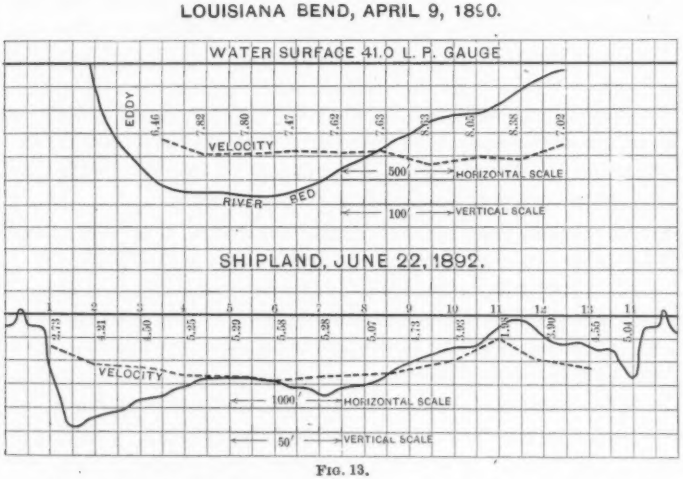


FIG. 13.

Mean Depth at any Particular Station not always Controlling.—The dictum that mean depth has an immediate controlling influence on velocity is frequently contradicted by the experience derived from measurements of relative velocity in various parts of the cross-section. It is true that the contradiction is more apparent than real, and that the variations of depth in such cases are often or always compensated by opposite changes of local slope. If the assumption that mean depth is controlling be absolutely true, it should hold good not only for different sections of a river, but for different parts of the same section. But it is found that the swiftest current is not always in the deepest water, even in the same section. This fact is clearly shown in the

plots of velocity observations. These are not usually published, but an instance may be cited, from sources accessible to examination, in the observations taken in Louisiana Bend in 1889, 1890 and 1891,* and at Shipland, in Mississippi, on June 21st and 22d, 1892.† Selections from these are reproduced in Fig. 13. In both these instances it will be observed that the strongest current is not uniformly or usually in the deepest water, even in flood time, but is sometimes in comparatively shallow depths.

In the former case, an explanation of this phenomenon is readily afforded by the knowledge that the local slope is widely different in different parts of the cross-section. The observations were taken at the extremity of a "point," in the bight of a deep bend, and most of them were taken at high stages. The slope next the bank in the deep water was very small at such times, in accordance with a general law. Across the point of the bar it was very steep. The rapid descent at the latter place more than compensated for the shallowness of the water. The reach at Shipland is broad and of slight curvature. It cannot be doubted, however, that, owing to local causes, the threads of the current that possessed the highest velocity had also a very steep local slope. The same phenomena are presented in the Helena observation of March 20th, 1885 (see page 410).

Complexity of Conditions.—It will be perceived that the complexity of local conditions may be such that it is almost impossible to unravel their intricacy or to deduce from them any general or average condition. What, for instance, is the local slope at Shipland or in Louisiana Bend? On the one bank it may be 0.5 or 0.6, or even 1, ft. to the mile; on the other it may be 0.05 ft.

The slope of the formula is the inclination of the stream at the point considered. This can hardly be regarded as a determinable quantity. Many attempts have been made to ascertain it by direct measurement, but without success. The fall per mile is so small that a trifling error in the levels will make a great error in the slope. It is very difficult to ascertain what is the exact water level, affected as it is by ebbs and flows, pulsations, etc., and it is well-nigh impossible to eliminate or to estimate the effects of eddies, cross-currents and threads of current. The local slope is not a simple

* "Report of Mississippi River Commission for 1893," Appendix 5, Plate 114.
† The same, Appendix 5, Plate 11.

matter at all. It has been shown already that there is reason to suppose that each part of the cross-section has a slope of its own, and further evidence will be adduced in support of the same opinion.

Velocity of Approach.—The doctrine of an established equilibrium, so generally accepted, has led to an almost total neglect of the consideration of the possible influence of the velocity of approach. After a state of equilibrium has supervened, this element is not to be considered at all. But if there be no such equilibrium, due account should be taken of it.

In the state of quasi-equilibrium, which may probably be considered the normal or usual condition of the river, velocity of approach often plays an important part in the adjustment of local conditions. The state of quasi-equilibrium consists, as has been stated, in a series of oscillations, of accelerations and checks. The general slope in flood time is too great for equilibrium; it is so great that, if equilibrium were established, the velocity necessary to bring it about would be fatal to the stability of the banks and bed. Therefore, whenever the river has a straight course for a few miles, its velocity is sensibly accelerated, until a corrective is applied by the deepening of the channel and the formation of a shoal or island below, by bends or by some other well known means.

It often happens that long pools occur, both at high and at low water, in which there is a strong current with less than the average slope. At high water, the mere appearance of a long pool with little slope may be deceptive; for at such a time the current is often not next the bank in the bend, but across points or bars, so that the bend approximates to the condition of a pool; but at low water the current hugs the bank closely. Now it is possible for a considerable velocity to be established at a low slope, but it is not possible that it should be acquired with no slope at all, or with a reverse slope. It is, however, possible that it may be maintained, once acquired, at such a slope under favorable circumstances for a limited distance.

Reverse Slopes.—In the series of observations made at Carrollton in 1879-80, under the direction of Captain Leach, and thoroughly discussed by him,* reverse slopes are mentioned as occurring persistently, and even at high water. Captain Leach does not consider it probable that this phenomenon could have been due to eddies, the latter, as he

* "Report of Mississippi River Commission for 1882," pp. 98-115.

remarks, being usually high-water phenomena, or being, at least, most vigorous at that stage and decreasing in activity with the decline of the river, whereas the reverse slopes were most decided at low stages. From the observations he draws the following important conclusions:

"But if it is admitted that the water was moving down stream, while at the same time its surface inclination was up stream, it must also be conceded that the momentum of the moving water enters as a factor in determining the conditions of its flow, and hence no exact relation necessarily or even probably exists, in natural channels, between the velocity of the water at a particular time and place and the local slope of its surface at the same time and place. This principle may be applied to the sectional areas as well. By the term local slope is meant the fall of the water surface which would result from a measurement made over a very short, or relatively infinitesimal, length. Such measurement is, in the nature of things, impossible, and the quantities here discussed are not strictly local slopes; but the argument turns on the existence of reverse slopes, which being demonstrated for any distance, it follows, from the continuity of the surface, that reverse slopes must exist over infinitesimal distances at numerous points between those determined.

"If local slopes be rejected, it follows that an average fall over an assumed distance, greater or less, must be taken, as representing the slope which determines the discharge."

The evidence of the existence of actual reverse slopes, however, cannot be considered indubitable. The surface of the river, as is well known, is a warped surface, and the current crosses from one side to the other, producing great disparities of slope on the two sides.

General and Local Slope.—In determining the coefficients, in the discharges of 1890, and, indeed, in all the reductions, the slope has been taken between the discharge station and the nearest gauge above and below. The slopes so ascertained have differed materially from one another as they were taken above or below the station. Thus the slope Bolivar-Arkansas City, in 1892, was 0.000044 when the slope Arkansas City-Greenville was 0.000063. The coefficient deduced from the one was 109 when that deduced from the other was 91. Nevertheless, there was a pretty regular relation between the two systems of coefficients. At Wilson's Point, in 1890, it was possible to get an approximation to a local slope by comparing the gauge at Lake Providence, about 10.5 miles distant. The results differ somewhat from both that deduced from Greenville, above, or Vicksburg, below, which are more accordant among themselves. They show the maintenance, with considerable

persistency of a local slope decidedly greater than the general slope either above or below. After the occurrence of a great crevasse below Lake Providence, the local slope increased considerably.

It is not possible that a velocity should be maintained indefinitely by momentum upon a slope which could not originate such a velocity. The resistance, R, whether the stream be in equilibrium or not, is proportional to plv^2; in other words it is the same for a given rubbing surface and a given velocity, irrespective of slope. The resistance will therefore be as great for a small slope as for a great one, provided the velocity be the same. The acceleration, other things being equal, is dependent on the slope. Therefore, under the supposed circumstances, there would be an acceleration which is less than the resistance, which would throw the stream out of equilibrium and cause a rise, contrary to the hypothesis. If the slope be too small to originate such a velocity as may be prevailing at any place, the resistance will gradually overcome it and lessen it.

The fact that a greater coefficient is required in the lower reaches of the river, where the slope is flatter, may be explained by admitting that the slope in this part of the valley is itself sufficient to confer and maintain the requisite velocity, under the favorable circumstances existing there, and that the coefficient of 150 or more required there is the normal coefficient for that form of section and trace of bed. In the upper reaches, where the conditions are very unfavorable, the velocity is reduced by shocks, bends and other circumstances which do not enter the formula, and the coefficient is less.

The distribution of the local slope in a transverse sense is shown by the published notes of the Helena discharge of March 20th, 1885.* Table No. III shows the partial areas, the observed velocity for each area, the partial discharges and partial mean depths. It shows also the partial coefficients, obtained by the application of the formula $v = c\sqrt{rs}$ to each partial area. For this purpose the slope will be assumed the same for all, being that actually prevailing on that day, as estimated from the nearest gauge station, Mhoon's Landing, 29.2 miles above. If, instead, the slope is taken from Helena to St. Louis Landing, 43 miles below, the only essential difference will be that the coefficients will be greater, but they will bear the same relation among themselves. The assumption that the slope is the same in all parts of

* "Report of the Mississippi River Commission for 1887," p. 2825.

the cross-section is more likely to be true at a medium stage, like that presented in the example, than at a very high stage; yet it is not at all correct even here, as will presently appear. In the table the areas are 200 ft. wide, except at the ends, where they are 261 and 126 ft. respectively. They are measured between half-stations, the velocities being taken at the whole stations, given in the table. Thus, the first area extends from Station ½ to Station 1½, plus 61 ft., etc. Areas are placed opposite the velocity stations, which are in their middle.

TABLE No. III.—HELENA DISCHARGE, MARCH 20TH, 1885.

Station.	Partial area. Square feet.	Velocity. Feet per second.	Partial discharge. Cubic feet per second.	Partial mean depth. Feet.	Partial coefficient.
1.....	9 084	3.51	31 885	34.8	64
2.....	12 415	4.30	53 385	62.1	68
3.....	11 945	4 97	59 367	59.7	69
4.....	10 637	4.86	51 696	53 2	71
5.....	10 090	5.00	50 450	50.4	75
6.....	9 570	5.10	48 807	47.9	79
7.....	9 420	5.12	48 230	47.1	80
8.....	8 598	5.05	43 420	43.0	82
9.....	7 720	6.01	46 397	38.6	104
10.....	6 982	5.32	37 144	34.9	96
11.....	6 140	5.26	32 296	30.7	102
12.....	5 570	5.13	28 574	27 8	104
13.....	5 065	4.22	21 374	25.3	90
14.....	4 940	4.30	21 242	24.7	91
15.....	4 890	3.72	18 191	24.5	93
16.....	4 978	4.14	20 609	24.9	80
17.....	4 940	3.88	19 167	24.7	84
18.....	4 815	3.82	18 393	24.1	83
19.....	4 902	3.80	18 628	24.5	82
20.....	5 045	3.50	17 658	25.2	75
21.....	5 320	3.19	16 971	26.6	66
22.....	5 343	3.05	16 296	26.7	63
23.....	4 875	2.95	14 116	23.9	64
24.....	1 461	1.54	2 250	11.6	48

Total area, 164 655 sq. ft.; velocity, 4.473 ft. per second; discharge, 736 546 cu. ft. per second; mean depth, 34.4 ft.; coefficient, 81.64.

From this table it is apparent that the coefficient increases very regularly, excepting one or two discordant observations, to the middle of the river, and then decreases, not quite so regularly, to the other bank. It seems to bear no relation whatever to the depth, and but little to the velocity. The coefficient at 3.5 ft. per second is the same as at 5 ft. It is probable, then, that the increase of the coefficient toward the middle of the stream depends on local slope. There is a "crossing" just below Helena, that is, a place where the current leaves one bank and crosses over to the other. It is well known that at such places there is often a great difference of elevation of the water surface

on the two sides, amounting, as has already been explained, to a considerable fraction of a foot, or even to a foot. It is probable that such a condition existed at this point, the thread of the current having a slope greatly in excess of the general slope of the river in this vicinity. Such instances are common, and mention has been made of other parallel cases.

Resistance at the Surface.—The formula of Humphreys and Abbot differs from all the rest in substituting for the hydraulic mean depth or hydraulic radius a "mean radius prime," obtained by dividing the area of the cross section by the wetted perimeter plus the width. This was because they found at the surface a resistance analogous to that experienced at the surface of contact between the water and the sides of the channel.

If the velocity in rivers were affected only by the friction of the bed, it is evident that in a straight and symmetrical channel the swiftest current would always be found in the parts most remote from the banks on one hand and from the bottom on the other. If the course of the stream were crooked and its cross-section unsymmetrical, the place of maximum velocity would no longer be in the middle, but it would still be at the surface. But it has long been known that the greatest current is not always or generally found in this situation. It is evident, then, that there is a force other than the friction of the bed which is at work, and this force exercises a retarding effect upon the water at the surface.

It is sometimes supposed and asserted that Humphreys and Abbot ascribed this retardation to friction against the air. This is altogether erroneous. They did indeed consider the resistance of the air as partly the cause of the retardation, but not wholly or mainly. The retardation was attributed by them principally "to the loss of living force arising from upward currents or transmitted motion occasioned by irregularities of the bottom. This loss is greater at the surface than near it."* So again:

"Enough has now been learned to justify the remark that the resistance at the surface in calm weather can be only partly due to the friction against the air; otherwise a down-stream wind, moving with equal velocity with the water, must reduce it to zero and raise the axis to the surface—a result contrary to the observations. It occasions no surprise, to one familiar to the boils and whirls of the Mississippi, that

* "Report on the Physics and Hydraulics of the Mississippi River," p. 302.

they should cause a great loss of living force at the surface, and consequently a great retardation of the surface current."*

The views thus advocated are substantially the same as those put forth afterwards by Professor James Thomson:†

"He points out that portions of water, with a diminished velocity from retardation by the sides or bottom, are thrown off in eddying masses and mingle with the rest of the stream. These eddying masses modify the velocity in all parts of the stream, but have their greatest influence at the free surface. Reaching the free surface they spread out and remain there, mingling with the water at that level and diminishing the velocity which would otherwise be found there."

It is a particular case of the reasoning previously quoted from Boussinesq.

All that was claimed by the famous American investigators was‡ that there was a retarding force of some sort at the surface, and "that this resistance, whatever its cause may be, is of the same order or nature as that at the bottom and sides of the channel, since the law of transmission through the fluid is the same in each case." That is, they held that the film of water next the air moved more slowly than the layers adjacent to it below, and hence exerted a retarding influence on them, and through them on the whole mass. The error they made was in attributing to this resistance the same importance as to the resistance of the bed, whereas it is not one-tenth of the latter. This error is not of great consequence. If there were absolutely no resistance at the surface, and the expression p, in the equations, for the wetted perimeter, were arbitrarily taken at double its real value, the area, a, remaining the same, it is obvious that the only effect on the resulting formula would be to reduce the value of the mean depth by one-half, and hence to increase the coefficient c.

Local Formulas.—In the face of these objections it would seem to be extremely difficult to arrive at a general formula adapted to channels of all sizes and conditions. It is very difficult to devise even a formula which shall be suited to large rivers, or to an individual river like the Mississippi, at all slopes, places and stages. But there is a chance of success in the search for local formulas, adapted to particular places and perhaps particular stages.

* "Report on the Physics and Hydraulics of the Mississippi River," p. 271, note.
† "Encyclopædia Britannica," Vol. XII, p. 497.
‡ "Report on the Physics and Hydraulics of the Mississippi River," p. 317.

General Considerations.

Some general considerations may be premised.

Nature of the Resisting Force.—The resisting force, if it be not in the shape of friction, is in the shape of work. Now, the resistance of a rough bank and bottom, so far as it is not purely frictional, is merely the interposition of obstacles to the free flow of the current. To overcome these obstacles requires work. It is not unanimously conceded that the resistance developed by a stationary obstacle to the motion of running water is the same as that presented by still water to the movement of a solid body through it. The preponderance of opinion, however, is very largely on that side. The identity or close analogy of the two cases is tacitly conceded when the results of Colonel Beaufoy's or of Mr. Froude's experiments are applied to the case of resistance of the banks. It is conceded whenever a current meter is rated by the usual method.

The resistance of a surface which is perpendicular to the direction of the current is approximately equal to the weight of a prism of water whose base is the surface and whose altitude is the height due to the velocity. The resistance of a surface which is inclined to the current is much less, according to the degree of inclination and also according to the form of the surface ; the coefficient of difference varying, in extreme cases, from 1 to 0.22. The water presses on both sides of the surface. In ordinary cases the difference of pressure on the upper and the lower side of a body, especially if it have both its faces inclined to the direction of the current, is so small that it may be neglected. In any event, the resistance is dependent on the square of the velocity and not on any other power.

The vertical and lateral disturbances in the nature of eddies and boils also are work, as well as all resistances except those of pure friction and viscosity, and possibly the latter also, if they be worthy to be taken into account. Hence it may be inferred that the resistances of all kinds are functions of the square of the velocity.

The velocity here mentioned is not the mean velocity, but is composed of velocities of all possible magnitudes between the maximum and that passing next the bottom. In the impossibility of finding any formula which shall express the relations between these elementary quantities of work, and consequently of the velocities on which they depend, it is necessary to rely on the mean velocity in hypothetical

cases, or on partial velocities so far as they may be ascertainable, regarded as means for the areas which they represent; trusting to correction, by empirical methods, of the errors introduced by this imperfection of method.

That the resistance of the bottom, other things being equal, varies with the extent of the rubbing surface, seems undeniable. That it varies in exact proportion to it, irrespective of shape or depth, cannot be asserted. Whatever errors are introduced by the assumption will show themselves in the coefficient.

Intricacies of Slope.—It seems to be impossible, with the available data, to unravel the intricacies of general and local slope, and the respective agencies of velocity of approach and of velocity due to fall. Usually nothing is known except the general slope. If the exact local slope were known for every filament of the surface, it is not certain that the investigation would be simpler. It is therefore necessary to accept the general slope for better, for worse, and make the most of it.

It will not be worth while to attempt to find a formula which shall be adapted to places and circumstances of both high and low slope, for the reason that there are important modifying conditions which are not cognizable by formulas at all, or, at least, by any which have ever been proposed. These conditions are the straightness or easy curvature of the river, symmetrical shape of cross-section, permanency of channel, smooth and mostly non-caving banks, etc. They prevail in the lower part of the river below Baton Rouge, and have a most important effect in modifying the velocity.

It is easily seen that in a straight and symmetrical channel, the banks and bottom of which are composed of unusually tough and hard material, the water will acquire a velocity much in excess of that which will prevail in a region full of sharp curves and with friable bed. Very little slope may be required to confer a considerable velocity where there are no obstructions. Now, the average curvature of the river is very nearly the same from Baton Rouge to Carrollton as from Natchez to Baton Rouge. The curves, however, are generally easier, the bottom is of harder material, the widths and depths more regular. There is not the same alternate expansion into shoals and contraction into pools as in the upper river. The river carries for the most part only its normal burden of silt, and its energy is not dissipated in transporting from place to place enormous loads of eroded

material beyond its ability to carry permanently. Now, the average curvature of two portions of a stream may be the same when the conformations, as to resistance, are widely different. Thus, in the two plans shown in Fig. 14 the ratio between the shortest distance, *A B* or *D E*, and the distance by river may be exactly the same; yet it is evident that the former trace will afford a far greater resistance to the passage of a current than the latter. Below New Orleans the river is almost straight, with the exception of the "English Turn."

The mere change of direction, if gradual, does not interfere with velocity. Thus the planets move in curved orbits without diminution of speed. It is the impact against resistances that causes the loss of energy, and hence of velocity; and this impact is more or less effective, according to the angle of presentation.

If the course of the river were straight, with a slope even as low as 0.1 ft. to the mile, the velocity would be more and more accelerated until it might become absolutely destructive to a bed composed of friable earth. In such a case it would scour out the bottom, produce an alternation of pools and shoals, and eventually bank erosion and the formation of bends. A bottom of more refractory material would allow of a higher velocity without suffering any alteration, and might even tolerate a current so swift as to bring about at that small inclination a condition of genuine equilibrium between the accelerating and retarding forces. In other words, it is perfectly possible for an inclination of 0.1 ft. to the mile at Carrollton to produce and maintain as great a velocity, under the circumstances existing at that point, as for an inclination of 0.4 or 0.5 ft. at Columbus or Helena under the peculiar conditions that characterize the river in those stretches. It is true that a part of the difference of circumstances appears in the ordinary formulas, under the form of mean depth; but only a part. The rest of the differences must appear, if any of those formulas be adopted, in the coefficient. As it is exceedingly difficult to devise a a general expression for the effect of the differences, it would appear

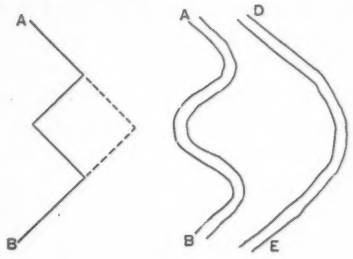


FIG. 14.

easiest and most satisfactory to find experimentally a coefficient for each place and perhaps a different coefficient for each stage.

Reduction by the Use of Coefficients.—This method is of constant adoption in the practical application of mechanics or even of mathematics. When the circumstances are so complicated as to defy the powers of analysis, a simpler case is assumed, the explanation of which is within reach, and the results of the rational formula deduced from the hypothetical case are compared with the phenomena shown by experiment. The two may bear a relation which shall be close enough to be expressed by a coefficient which is either constant, or variable according to a known law. If a series of parabolic areas are to be computed and the method of calculating parabolic segments is not known, in default of better methods they may be assumed to be rectangles. If the contents of one of the given areas is measured, by dividing it into partial areas or otherwise, the relation between it and the corresponding assumed area is found to be two-thirds, and this expression, applied as a coefficient to each of the assumed areas, will reduce it to the correct quantity. Most often the result is not so satisfactory, and frequently it is necessary to be content with a mere approximation and often with a coefficient that varies according to laws which are themselves obscure.

Causes of Discrepancies in Actual Velocities.

The great discrepancies found in the measurements of velocity are principally due to these four causes: (1) inaccuracy of data; (2) inapplicability of formulas; (3) changes of slope; (4) changes of cross-section; the last class being partly connected with the preceding or third class.

The first and second causes have been sufficiently discussed. Changes of slope are of several kinds. There are: (1) changes due to rise and fall; (2) changes due to stage of river; (3) changes due to alteration of section by scour and fill during the season; (4) changes due to permanent change of bed by secular causes.

The departures of the discharge curve from the regular figure which it is generally expected to present almost all exhibit considerable persistency, indicating the existence of law. During the progress of a rise it was observed that the curve uniformly shows a departure to the right, indicating a decided increase, and as decided a shifting to the

left or lower side during a fall. By comparing the curve of discharges with the curve of velocities (see Plate VI), it will be found that the principal features of the former are faithfully reproduced in the latter, while the variations of the area move sometimes in a direct and sometimes in an inverse sense.

Of the more regular movements of the velocity, the greater part are attributable to change of slope; the rest mostly to change of area.

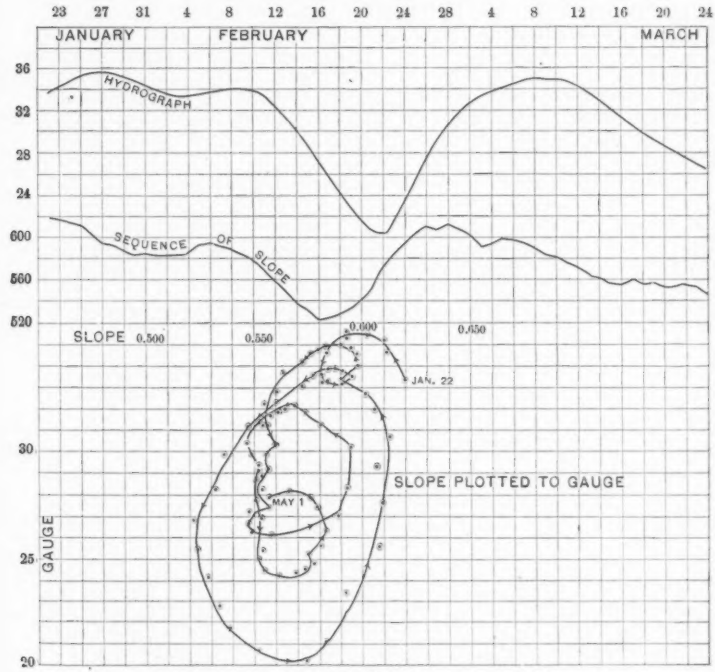


FIG. 15.

I. Changes of Slope.—Each accession of water from above, at any station, from the main river or from a tributary, causes an elevation of the surface at that station, the influence of which, at first, is confined to a limited distance. In other words, the rise at *A* is at first absolutely imperceptible at *B*. If the freshet be sudden and the stations 100 miles or so apart, there may be a rise of several feet at the upper station while the lower is unaffected. The slope, as estimated between these

stations, will then experience a sudden and great augmentation, which may progress at a still more rapid rate, may continue at the same rate, or may fall off. Sooner or later, all these alternations will take place in the course of every rise. The process which goes on during the fall is just the reverse of this.

The movement of the slope in rises and falls is shown in Fig. 15. taken from the gauge records of 1889. The points selected for comparison are White River Landing and Arkansas City, 43.9 miles apart. The first plot shows the movement of the slope from day to day, or what Colonel Suter calls a sequence of slope. The second shows the movement with reference to the gauge.

From both plottings it will be seen that in this distance there is no regular relation between the height of the river and the slope. The actual value of the slope evidently depends on the suddenness and magnitude of the freshet. This particular slope is much more liable to disturbance than most others, as White River Landing is just below the mouth of two large tributaries, with drainage basins which are the recipients of a large rainfall, often very fitful.

Effect of the Great Basins on Slope.—At high water, the fluctuations of slope are often very great from the disturbing influence of the great basins in subtracting water from the river at their upper ends and returning it at their lower ends. In former years this influence was much greater than it is at present. When the upper parts of the Yazoo and Tensas Basins and all of the Saint Francis and White River Basins were open, as they were in 1882, 1883 and 1884, as much as 300 000 or 400 000 cu. ft. per second were suddenly added to the discharge at Helena, Vicksburg and Red River Landing, raising the river with great rapidity at those points, greatly flattening the slope above them and steepening the slope below. In 1882, when the water which escaped over banks and through gaps and crevasses into the Yazoo Basin returned at Vicksburg, the river, which was then falling, began to rise rapidly (0.5 ft. per day); the slope from Hays's Landing to Vicksburg, which had been 0.27 ft. to the mile, dropped to 0.18 ft. and a less discharge passed Hays's Landing at 38.6 ft. on the gauge than had passed previously at 36.4 ft., or than passed afterwards at 34 ft.

Peculiarity of the White River Basin.—With the complete closure of the Yazoo and Tensas Basins, and the approaching closure of the other two, these perturbations will be reduced to such as are produced by the

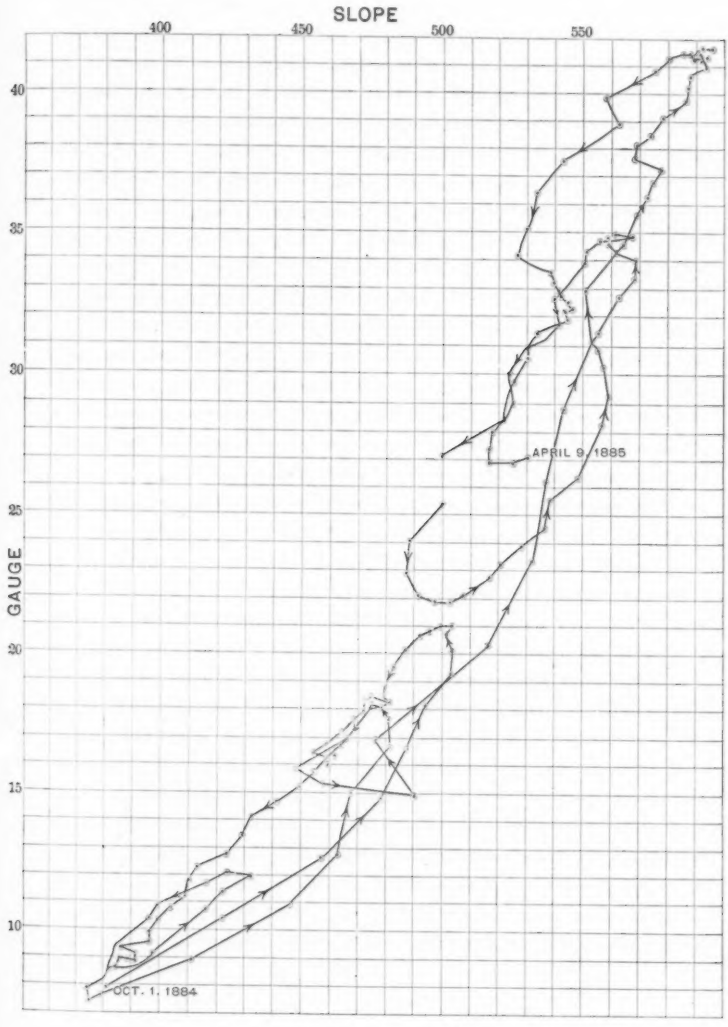

SLOPE
400
450
500
550
40
35
30
25
20
15
10
GAUGE
APRIL 9, 1885
OCT. 1, 1884

FIG. 16.

actual discharge of the tributaries, increased occasionally by the discharge from great crevasses, should such unfortunately occur. The condition of the White River Basin is peculiar in this respect, that at high stages a considerable part of the discharge of the two great tributaries is not poured directly into the Mississippi through the mouths of the White and Arkansas Rivers, but overflows the basin, flows through it parallel to the main river, and is discharged along the Cypress Creek levee, a few miles above Arkansas City, thus introducing a new element of disturbance into the slope relations.

Slope Arkansas City-Greenville.—Between Arkansas City and Greenville there is no disturbing influence, unless, perhaps, at high water a crevasse might occur. The gauges are 40 miles apart. A plotting is given in Fig. 16 of the curve of slopes between these stations, with the gauge height at Arkansas City as the argument, for the season of 1884-85. Here there is seen an increase of slope with height of river. The slope is much more regular than the slope White River-Arkansas City.

Movement of Gauges.—The increase of slope with gauge height, however, is not a feature which prevails everywhere. Indeed it cannot be, for such a relation would involve a progressive decrease of oscillation from the head of the alluvial valley downward, which is not true. The range at Cairo is about 1 ft. less than at Vicksburg, and a little more than at Arkansas City. The range at Greenville and Lake Providence is some 7 ft. less than at Cairo, and 8 ft. less than at Vicksburg. The gauges at Arkansas City and Vicksburg move very nearly parallel, so that if the slope Arkansas City-Lake Providence increases with a rising river, the slope Lake Providence-Vicksburg must diminish. Such in fact is the case. The following little table shows the extremes, taken from the high water of 1893, the highest recorded at Arkansas City, and the low water of 1894, the lowest ever known throughout this reach. As there were several crevasses in 1893, a correction has been applied to the readings of the Greenville, Lake Providence and Vicksburg gauges, to deduce from them the water level as it would have been if those disturbing influences had not occurred. For comparison, the high-water slope of 1888 is also given. In that year there were no crevasses.

Slope.	Low water, 1894.	High water, 1893.	High water, 1888.
Arkansas City-Greenville	0.178	0.340	0.323
Greenville-Lake Providence	0.332	0.322	0.327
Lake Providence-Vicksburg	0.437	0.302	0.307
Arkansas City-Vicksburg	0.331	0.319	0.319

In other parts of the river it is found that the same discrepancy prevails, the stations between Cairo and Helena showing a much less oscillation between high and low water than prevails at those points. At Memphis it is 16.2 ft. less than at Cairo, and 13.3 ft. less than at Helena.

Greater Oscillation at the Mouths of Tributaries.—It has long been known* that there was a much greater oscillation of the gauges at the mouths of tributaries than at intermediate stations. This is partly due to the greater discharge at the former places during great floods. For instance, at Cairo, the greatest discharge that was passed between banks in 1882, as reported, was about 1 500 000; at Fulton, about 1 250 000; at Memphis, about 1 250 000; at Helena, about 1 500 000; at Arkansas City, about 1 300 000; at Greenville, about 1 200 000; at Lake Providence, about 1 000 000; at Vicksburg again, about 1 500 000. These differences proceeded from loss of volume over banks unprotected by levees and through crevasses in the latter, and from the return of the water thus lost at the foot of the several basins. Had as great a discharge passed Memphis as passed Cairo, the water level at the former station might have been 4 or 5 ft. higher. This, however, only accounts for a part of the discrepancy. In 1885, when there was no overflow, the oscillation was, at Cairo, 30.7; at Fulton, 23.7; at Memphis, 25.1; at Helena, 32.7; at Arkansas City, 34.9; at Greenville, 29.7; at Lake Providence, 29.1; at Vicksburg, 37.5. The greater oscillation at Helena, Arkansas City and Vicksburg than at Cairo is partly explained by the greater volume of water which passed at those points by the accession of tributaries, growing continually larger as progress is made down stream, and sometimes reduced to almost nothing when the tributaries were very low; but the difference between Arkansas City and Greenville or Lake Providence, or the difference between Cairo and Fulton, cannot be thus explained. As has been shown in the "Report of the Mississippi River Commission for 1884,"† the difference seems here to be in the low-water mark; it is in the extreme fall and not in the extreme rise.

The proximate cause of the less oscillation of the river at intermediate stations is the profile of the river-bed, which is convex between the mouths of tributaries. Now the low-water slope conforms to the

* "Report of the Mississippi River Commission for 1884," p. 2550.
† p. 2550.

shape of the bed much more closely than does the high-water slope. The former, then, is flattened in the reaches between the mouths of tributaries and the intermediate stations, involving as an immediate consequence a decrease of velocities at these points and an increase of cross-section at low water. The data for the comparison are scanty, for not many low-water discharges have been taken. At Helena, in 1891, the extreme range of recorded velocities was from 6.47 to 2.24. At Fulton no extreme high-water discharge was taken. The highest recorded velocity was 7.01; the low-water velocity was as small as 1.51. At Memphis, the velocity varied from 8.65 to 1.40; at Wilson's Point, from 5.83 to 1.84; at Red River Landing, from 5.24 to 2.63.

The most striking example of a small oscillation is afforded by Carrollton, which is a suburb of New Orleans, situated at its upper limit. Here the extreme oscillation is only about 17.4 ft. The high-water velocity is usually something more than 6 ft. Very few low-water observations have been taken at Carrollton; but in 1891, the lowest velocity must have very little exceeded 1 ft. per second. The water area generally varies from about 190 000 at high water to 140 000 at low. In 1884–85, its extreme limits were 185 072 and 148 437; the velocity varying from 6.16 to 1.61. At Helena, during the same season, the areas ranged from 206 060 to 53 448, and the velocities from 4.96 to 2.82. Thus the discharge is passed at the lower station with a much less difference of cross-section between high and low water, and consequently with a much less oscillation in height. In the above instance the range of the gauge at Helena was from 40.62 to 7.17 = 33.45 ft.; at Carrollton, from 13.54 to 0.34 = 13.20 ft. The comparison is still more striking if the case of Red River Landing is taken, only 192 miles above Carrollton, and with exactly the same discharge. In 1884–85, the areas at Red River Landing ranged from 236 688 to 97 589, and the velocities from 5.08 to 2.25. The oscillation was 34.58.

It has been shown that the slope from Arkansas City to Greenville varies from 0.178 at low water to 0.340 at high. On the other hand, the slope from White River to Arkansas City at low water is 0.378; at high water 0.269 ft. to the mile. Thus, at Greenville the conditions are favorable to a comparatively low velocity at low water and a high velocity at high water, while at Arkansas City the tendency is in the opposite direction.

The Beds of Sedimentary Streams Self-Regulating.—The phenomenon of the convexity of the river-bed between tributaries is so general that it assumes the proportions of a law, and it demands an explanation. A plausible one is found, in the alluvial part of the river, in the effects of the variations of discharge alluded to above. It is regarded as a well established fact that the bed of a sedimentary river regulates itself according to the quantity of water it has to carry. Now, at the mouths of tributaries the discharge is much greater than at the intermediate points. The great volume here poured in has scoured out a suitable channel for itself, which has been gradually contracted down stream by the continual loss of water in floods over the banks. In the case of the Mississippi, the discharge of the tributaries has been greatly augmented, sometimes doubled, trebled or quadrupled, by the return flow from the great basins. Usually, the tributary which drains the basin is the principal channel through which the overflow water returns to the main stream, as in the cases of the Saint Francis and the Yazoo Rivers. The White and Arkansas Rivers, as has been noted, form an exception to this rule.

Causes of Contraction of Bed by Loss of Volume.—It is not necessary to discuss in detail in this place the process by which the bed is contracted by the loss of volume of water. In brief, the contraction is due to the slackening of velocity brought about by the disproportionate reduction of area in comparison with wetted perimeter, or in short by a diminution of mean depth. In the equation $r = \frac{a}{p}$, a is reduced more than p. A decrease of gauge height of 8 ft. means a reduction of wetted perimeter of only 16 ft., or 0.004, but of area of 32 000 sq. ft., or 0.133 of the whole. No doubt also some consequence is to be attached to the deflection of the thread of the current by reason of the lateral draught, though the degree of importance to be attached to this influence is still a matter of controversy.

The oscillations of slope due to stage must not be confounded with those caused by rise and fall. The latter are temporary disturbances of the ordinary relations, and may occur at any time. The former are of a permanent nature, and while not perfectly regular in their movements, yet, when dissociated from the perturbations of rises and falls, they exhibit a general law of direct or inverse progression relatively to the stage. This progression can be exam-

ined in the tables of discharges which have been selected for discussion, and in the plottings.

The table given on page 420 may be extended as follows, from the observations of 1891. This year was selected because it was not a year of very great flood nor therefore of great disturbance of high-water conditions, and also because it was a year of very low water.

Slope.	Low-Water Slope, 1891.	High-Water Slope, 1891.
Cairo-Columbus	0.196	0.310
Columbus-New Madrid	0.219	0.341
New Madrid Cottonwood Point	0.521	0.476
Cottonwood Point-Fulton	0.362	0.422
Fulton-Memphis	0.482	0.432
Memphis-Mhoon's Landing	0.439	0.486
Mhoon's Landing-Helena	0.547	0.328
Helena-Sunflower Landing	0.295	0.405
Sunflower-Mouth of White River	0.375	0.277
White River-Arkansas City	0.378	0.269

As has been stated, these movements are not at all regular. Sometimes, at low water, the slope White River-Arkansas City will be less than at high, as in 1884–85 and in 1889. At all times considerable perturbations are likely to occur in the transitions from stage to stage, from changes of cross-section, etc.

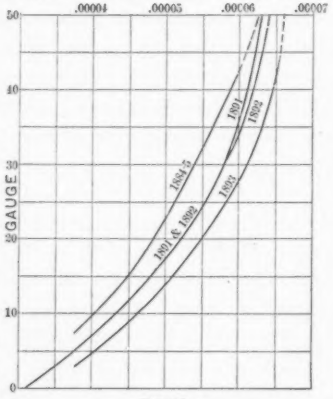


FIG. 17.

Secular Changes of Slope.—Beside these temporary or periodic changes of slope there are others that may be called secular, requiring a considerable time for their development, and of a permanent character. They are probably intimately connected with changes of the bed. Observations on the Mississippi have been conducted for too short a time to afford grounds for any accurate conclusions as to the effect of long-continued causes, enduring for a series of years, on the regimen of the river. It is probable, however, that the slope relations between different points have been materially modified since the beginning of the observations. A sketch is given in Fig. 17, showing the slope Arkansas City-Greenville, plotted to gauge, for the years

1884–85, 1891, 1892 and 1893, on the assumption that the distance between the two points remained constant. In order to eliminate the effect of rises and falls, the slopes have been taken only at times when the river was stationary, or at the culmination of a rise or the lowest point of a fall. It is to be observed that in many instances the change was so rapid that it is by no means certain that the exact slope at such times was ascertained. The observations are taken but once a day and generally to the nearest tenth of a foot, and on successive days the slopes are sometimes found to vary by 0.01 ft. to the mile, or 0.000002 of the distance.

Changes of Slope between Arkansas City and Vicksburg.—It would appear by an inspection of Fig. 17 that the slope Arkansas City-Greenville had increased materially in eight years, and by a steady progression applying as well to the low as the high stages. This phenomenon may indicate an enlargement of the cross-section at Greenville. It must be remembered, however, that the length of the river is perpetually changing. Accurate data as to the amount of the change are not forthcoming, but it is probable that the distance between Arkansas City and Greenville has increased since 1882 by about 2 miles. If this be true, the slope, in this instance, has really suffered little or no alteration.

II. Changes of Area.—The changes which the area undergoes are at least of three kinds. One is the daily alteration caused by the passage of sand waves, the caving of banks, and generally by incidents which may be considered of a temporary and fortuitous nature. Another is the grand annual periodic change due to the transition from high to low water or the reverse, whereby material is transferred from bar to pool, or from pool to bar, the general or mean bed of the river suffering no change. A third embraces changes which are permanent or of long period, such as those just alluded to.

Rapid Fluctuations of Area.—The daily fluctuations of the area are sometimes extremely violent. At Arkansas City, in 1885, a scour of 11 000 ft. in two days and of 14 000 ft. in three days is recorded, followed by a fill in the next two days of 8 000 ft. Changes of 5 000 or 6 000 ft. in a day are not uncommon. When it is realized that a mistake in sounding of a foot or two throughout the section would produce such an apparent result, there is a natural feeling of incredulity when these figures are read. The observers, however, who are usually educated

and skilful engineers, believe firmly in their reality. In 1892, at Wilson's Point, Mr. Miller reports, from Stations No. 4 to 6, where very large boils existed, a fill of 16 ft. in 24 hours.

It has already been remarked that these changes of area are generally accompanied by a change of velocity, and that this is a strong confirmation of their genuineness. There is no reason whatever why an error in sounding should have any effect on the indications of a current meter. If, therefore, a mistake were made in the measurement of a cross-section, the result would be perceptible in the shape of an increased or diminished discharge. If there be no such result, but the change of area is offset by a change of velocity in the opposite direction, the presumption is in favor of the reality of the phenomenon.

Passage of Sand Waves.—A number of very careful experiments have been made to determine the quantities of material moved along the bottom, or rolled in the form of sand waves, as they are generally called, and to measure the height, shape and direction of the waves, their rate of travel, etc., at Plum Point, Lake Providence and Carrollton during the season of 1879–1880. It would appear that these movements are sometimes of great magnitude, the crests of the waves being often as high as 22 ft., their slopes flat on the upper side, and steep on the lower. In 1880, Arthur Hider, M. Am. Soc. C. E., found that at high water, in the shallow sections, the average height was about 13 ft., the distance between crests about 450 ft., the rate of travel about 22 ft. a day. In the deep sections, the height of the waves at high water was about 7 ft.; length, 330 ft.; travel, 40 ft. a day. At lower stages the waves are not usually so high, the distances are shorter and the rate of travel slower; but the movements are still of considerable magnitude. Even in very shallow water there were found numberless small sand waves 1 ft. or 2 ft. high and less than 100 ft. apart. Now, the passage of a wave 1 ft. high would make a difference of area, in an ordinary discharge section, of 3 000 or 4 000 sq. ft.

Changes of Area Due to Alternations of Stage.—By far the most important changes of area, however, are those which take place during the transitions from high to low stages and the reverse; and it is in these changes that some of the most curious and characteristic phenomena of the river find their explanation.

Of the many varieties of cross-section found in the Mississippi as well as in other rivers, there are two extreme types, the narrow and the broad, pertaining respectively to the pools and the shoals. At

medium stages these do not differ very much in area. At low water the pools have much the larger section, at high water the bars. Let Figs. 18 and 19 represent two such sections. At a medium stage the two will have about an equal content, say, 180 000 sq. ft. Now let the depth be diminished by 10 ft. The width of the broad section is twice as great as of the narrow. It will therefore lose, perhaps, 65 000 sq. ft. of area, while the narrow section loses only 32 500 sq. ft. Will a given decrease of discharge produce a fall of equal amount at the two sections, widely diverse in form as they are? Experience shows that at two stations, only a few miles apart, the rate of rise or fall is nearly uniform, irrespective of the shape of the section. Reasons will be given hereafter why this must be so.

Now the discharge that passes any station does not depend in the least upon the slope, depth or condition of the bed at that place. It depends solely upon the quantity of water furnished from above. While the supply continues the same, the discharge is constant. If, then, the area be increased, the velocity must be decreased. Either this, or the water surface must suffer a fall sufficient to compensate the increase of area. Experience shows that no such fall occurs, and reason asserts that it could not occur. On January 9th and 10th, 1885, at Arkansas City, a scour of 11 237 ft. is recorded. The width of the river was 3 370 ft. If the same velocity had been maintained after the change of area as had existed before it, a fall would have occurred of 3.3 ft. Instead, the rise which was steadily progressing at the rate of 0.5 ft. per day was only slightly disturbed. Indeed, in a stretch of river which has a mean fall of only 0.32 ft. to the mile, it is evident that no serious change of level can take place without disturbing the whole stream. That there was a slight change of level is evident from an inspection of the slope curve in Fig. 16, where a palpable disturbance is shown at about 38 ft. The change was only 0.1 or 0.2 ft. This is a striking proof of the reality of the fluctuations of area.

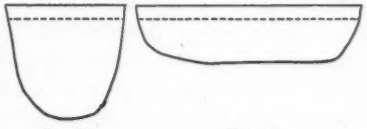


FIG. 18. FIG. 19.

It is not possible, however, that the velocity should diminish while the slope remains unaltered and the depth increases. The local slope must therefore accommodate itself to the change of depth and velocity. A change of slope of 0.048 ft. to the mile would compensate for

an increase in depth of 3.3 ft. and a decrease of velocity of 0.3. This change, of course, is maintained over only a short distance.

So long as the cross-sections are equal, the velocities will be equal also. As soon as the wide cross-section becomes less than the narrow, the velocity of the former will become greater than that of the latter, and this disparity will increase as long as the downward movement of the river continues. If the bed of the river were unalterable, the ultimate result would be that the current over the bars would approximate to the condition of a cascade or rapid, while that in the pools would be comparatively stagnant.

The bed of the river, however, is far from unalterable; the effect of the increased velocity over the bars in time of low water is to produce a scour, while that of the diminished velocity in the pools is to promote a deposit of the suspended matter brought down from the bar above. During a rise, the cross-section over the bar increases more rapidly than that in the pool and soon surpasses it in area. Its velocity then diminishes as compared with that of the narrow section. Soon the current in the wide section becomes slack, while that of the pool keeps increasing in rapidity. The latter then becomes the scene of scour, while at the former a fill is taking place. Hence, the well known phenomenon, which is regularly repeated every season, of the transference of enormous quantities of material from place to place with every great change of stage. These movements are of great magnitude, the datum cross-section nearly always undergoing alterations in a single season of 25 000 to 30 000 sq. ft.*

Secular Changes of Area.—The secular or permanent changes of area have already been alluded to, and Greenville and Wilson's Point have been cited as examples of what seems a permanent enlargement of section. So far as Greenville is concerned, this fact cannot be established by direct measurement, for no discharge observations have been taken there. At Wilson's Point there is satisfactory evidence that the cross-section has increased in area since 1890 by about 30 000 sq. ft. The greater part of this enlargement of area has taken place above the low-water line, as Captain Townsend has remarked.† Whereas the whole section has experienced the augmentation mentioned above, the low-water area seems to have increased only about 5 000 sq. ft.

* These points are well elucidated in a paper "On the Protection of the Lower Mississippi Valley from Overflow," by J. B. Johnson, M. Am. Soc. C. E., in the *Journal* of the Association of Engineering Societies, Vol. III, No. 9.

† "Report of Mississippi River Commission for 1893," p. 3762.

It would, perhaps, be possible to trace out the changes of the river-bed in other sections by a careful study of the slopes. The data for such a study are abundant, and are valuable for their continuity and their accuracy. They have been accumulating for 24 years; they are certain and exact within very narrow limits. There is no room for any considerable error in them. Such a study, however, would be laborious, and the conclusions would have to be freed carefully from perturbations due to other causes than change of area. It is hardly likely that it will be undertaken.

It is highly probable that a series of years of unusually high or unusually low water will leave its impress on the bed of the river. In fact, there is good evidence that such is the case.

Discussion of the Discharge Observations of 1884-85 at Arkansas City.

With this preparation for the investigation, an attempt will be made to develop some law of coefficients of the ordinary formula, using the general slope and the mean depth. For this purpose the discharge observations made at Arkansas City in 1884-85 will be used.* This place is selected because it is situated at the head of a reach wholly confined by levees, and below the confluence of the two greatest tributaries of the lower Mississippi; and because there is a station 93 miles below, Wilson's Point, where discharges have also been taken for several years, thus affording a valuable check on the measurements made at the upper station. The observations of 1884-85 are the first ever taken at Arkansas City. In the discussion by Mr. Wheeler, several sources of error are pointed out. Prior to October 12th observations were made with the meter at not more than 12 ft. below the surface. No corrections have been applied to reduce them to mean velocity. It will be seen that the coefficients deduced from them are much too great. Subsequently to October 12th velocities were taken at 0.6 of the depth, and the results accepted as the mean velocities in those verticals. There is room for error in the areas, owing to large changes in the lead-line, but, as it is unknown where they occurred, it is useless to attempt to correct them.

Table No. IV contains the principal data of the discharge observations of 1884-85, together with the slope, as estimated from the gauge records, from Arkansas City to Greenville, the theoretical velocity

* "Report of the Mississippi River Commission for 1887," pp. 2835-40,

TABLE No. IV.—Discharge Observations at Arkansas City, Ark.

Date.	Gauge reading.	Dimensions of Cross-Section of Discharge.					Mean velocity per second.	Discharge per second.	Slope, Arkansas City-Greenville.	Theoretical velocity.	Coefficient.	Direction and force of wind.
		Area.		Mean depth.	Width.	Scour or fill.						
		Water.	Below datum.									
1884.	Feet.	Sq. ft.	Sq. ft.	Feet.	Feet.	Sq. ft.	Feet.	Cu. ft.				
October 1	7.80	.....	.....	.....	.....	.....	.....	.....	0.00003708	.....	.....	
2	8.70	.....	.....	.....	.....	.....	.....	.....	0.00003897	.....	.....	
3	10.40	.....	.....	.....	.....	.....	.....	.....	0.00004217	.....	.....	
4	12.50	.....	.....	.....	.....	.....	.....	.....	0.00004539	.....	.....	
5	14.80	.....	.....	.....	.....	.....	.....	.....	0.00004791	.....	.....	
6*	16.60	139 391	223 301	42.5	3 282	.....	3.263	454 778	0.00004876	4.55	71.7	
7*	18.05	145 106	224 247	44.2	3 285	+ 946	3.344	485 275	0.00004937	4.67	71.6	
8	19.25	153 954	229 149	46.7	3 298	+4 902	3.472	534 514	0.00005042	4.85	71.6	
9*	20.00	149 885	222 606	45.4	3 302	−6 543	3.594	538 721	0.00005042	4.78	75.1	
10*	20.60	159 929	230 664	48.4	3 302	+8 058	3.449	551 566	0.00005009	4.92	70.1	
11	21.00	.....	.....	.....	.....	.....	.....	.....	0.00005042	.....	.....	
12	20.65	.....	.....	.....	.....	.....	.....	.....	0.00004923	.....	.....	
13	20.15	153 144	225 370	46.4	3 299	−5 294	3.149	482 228	0.00004862	4.75	66.3	
14	19.50	150 089	224 469	45.5	3 299	− 910	2.977	446 854	0.00004820	4.68	63.6	W.
15	18.85	145 259	221 773	44.1	3 295	−2 088	2.915	428 418	0.00004796	4.60	63.4	Calm.
16	18.40	143 698	221 695	43.6	3 295	− 77	2.826	406 063	0.00004791	4.57	61.8	Do.
17	18.15	143 533	222 404	43.6	3 295	+ 709	2 972	426 741	0.00004749	4.55	65.3	
18	18.10	144 477	223 463	43.8	3 295	+1 059	2,863	413 667	0.00004749	4.56	62.8	Do.
19	18.25	.....	.....	.....	.....	.....	.....	.....	0.00004782	.....	.....	
20	18.47	144 501	222 268	43.8	3 296	−1 195	2.806	405 416	0.00004744	4.58	61.2	N.E., light.
21	18.50	144 034	221 702	43.7	3 296	− 566	2.907	418 640	0.00004712	4.54	64.1	S.E., light.
22	18.42	143 372	221 304	43.5	3 295	− 398	2.890	414 295	0.00004744	4.54	63.6	
23	18.20	141 900	220 556	43.1	3 295	− 748	2.943	417 592	0.00004726	4.51	65.2	W., strong.
24	17.95	139 656	219 137	42.4	3 295	−1 419	2.823	394 222	0.00004721	4.47	63.1	E., light.
25	17.55	138 468	219 237	42.1	3 286	+ 120	2,710	375 263	0.00004688	4.44	61.0	
26	17.10	.....	.....	.....	.....	.....	.....	.....	0.00004650	.....	.....	
27	16.75	135 446	218 861	41.3	3 281	− 396	2,773	375 631	0.00004580	4.35	64.0	Calm.
28	16.43	133 525	217 990	40.7	3 279	− 871	2,676	357 325	0.00004547	4.30	62.2	W., strong.
29	16.20	131 501	216 718	40.1	3 277	−1 272	2.691	353 917	0.00004571	4.28	62.9	Calm.

	30	16.05	130 136	215 847	39.7	3 277	— 871	2.707	352 295	0.00004594	4.27	63.4	Do.
	31	16.30	132 444	217 334	40.4	3 278	+1 487	2.660	352 249	0.00004608	4.31	61.7	Do.
November	1	16.50	134 536	218 775	41.0	3 282	+1 441	2.723	366 394	0.00004641	4.36	62.4	Do.
	2	16.60								0.00004641			
	3	16.40	131 074	215 643	39.9	3 282	−3 132	2.688	352 363	0.00004608	4.29	62.7	Do.
	4†	16.25	130 370	215 430	39.7	3 281	— 213	2.750	358 517	0.00004594	4.27	64.4	N. W., strong.
	5	16.10	130 578	216 126	39.8	3 278	+ 696	2.774	362 230	0.00004580	4.27	64.9	N. W., light.
	6	16 00	130 875	216 750	39.9	3 277	+ 624	2 741	358 775	0.00004571	4.27	64.2	W.
	7	15.70	129 375	216 232	39.5	3 275	— 518	2.669	345 291	0.00004547	4.24	63.0	Calm.
	8	15.20	126 492	214 931	38.7	3 270	−1 251	2 673	338 146	0.00004509	4.17	64.1	Do.
	9	14.65								0.09004406			
	10	14.00	121 535	213 951	37.2	3 265	−1 030	2.575	312 907	0.00004311	4.00	64.3	Do.
	11	13 35	120 405	214 938	36.9	3 260	+ 987	2.486	299 275	0.00004297	3.98	62.4	Do.
	12	12.75	118 694	215 177	36.5	3 255	+ 239	2.496	296 235	0.00004241	3.93	63.4	Do.
	13	12.25	115 843	213 949	35.6	3 251	−1 228	2.460	284 955	0.00004123	3.83	64.2	Do.
	14	11.70	113 466	213 361	34.9	3 248	— 588	2.370	268 895	0.00004100	3.78	62.6	Do.
	15	11.18	112 829	214 413	34.8	3 245	+1 052	2.377	268 251	0.00004085	3.77	63 0	Do.
	16	10.70								0 00004029			
	17	10.25	108 334	212 859	33.6	3 224	−1 554	2.349	254 452	0.00003991	3.66	64.1	Do.
	18	9.85	107 053	212 851	33.8	3 165	— 8	2.270	243 034	0.00003959	3.66	62.1	N. W., light.
	19	9.50	105 972	212 881	33.6	3 154	+ 30	2.472	261 932	0.00003959	3.65	67.8	Do.
	20	9.25	104 764	212 462	33.6	3 119	— 419	2.298	240 774	0.00003865	3.60	63.8	Calm.
	21	9.05	103 376	211 708	33.1	3 119	— 754	2.419	250 087	0.00003921	3.60	67.1	S. E., light.
	22	8.93	103 916	212 628	33.3	3 119	+ 920	2.435	253 004	0.00003897	3.60	67.6	E., strong.
	23	9.00								0.00003912			
	24	8.90	103 042	211 849	33.0	3 119	— 779	2.521	259 781	0.00003902	3.59	70.4	Calm.
	25	8.83	103 046	212 075	33.0	3 119	+ 226	2.357	242 859	0.00003865	3.57	66.0	S. W., light.
	26	8 67	102 456	212 002	32 9	3 118	— 73	2.399	245 850	0.00003865	3.56	67.3	Do.
	27	8.58	101 707	211 526	32.7	3 115	— 476	2.342	238 191	0.00003865	3.55	65.9	Do.
	28	8.50								0.00003817			
	29	8.66	101 842	211 407	32.7	3 115	— 119	2.362	240 522	0.00003832	3.54	66.7	
	30	9.10								0.00003973			
December	1	9.77	105 412	211 466	33.4	3 154	+ 59	2.454	268 684	0.00004048	3.68	66.8	S., light.
	2	10.66	107 216	210 427	32.2	3 220	−1 039	2.637	282 728	0.00004161	3.66	72.0	Calm.
	3	11.38	110 268	211 198	34.0	3 245	+ 771	2.609	287 696	0.00004170	3.76	69.3	Do.
	4	11.89	111 773	211 046	34.4	3 248	— 152	2.617	292 500	0.00004320	3.85	67.9	S., strong.
	5	12.00	111 208	210 129	34.2	3 250	— 917	2.581	287 037	0.00004241	3.81	67.8	S. E., strong.
	6*	11.60	110 719	210 942	34.1	3 248	+ 813	2.894	320 386	0.00004161	3.77	77.3	S., very strong.
	7	10.85								0.00003996			
	8	10.32	106 816	211 137	33.1	3 229	+ 195	2.434	259 964	0.00003921	3.60	67.4	Do.
	9	9.55	103 542	210 297	32.8	3 154	— 840	2.431	251 696	0.00003874	3.56	68.2	Do.
	10	8.80	100 444	209 528	32.2	1 119	— 769	2 465	247 521	0.00003827	3.50	70.2	Calm.
	11*	8.22	98 290	209 249	31.6	3,115	— 279	2,626	258 131	0.00003779	3.47	74.2	S., strong.

* Reject. † Water rough.

TABLE No. IV—DISCHARGE OBSERVATIONS AT ARKANSAS CITY, ARK.—(*Continued*).

Date.	Gauge reading.	Dimensions of Cross-Section of Discharge.					Mean velocity per second.	Discharge per second.	Slope, Arkansas City-Greenville.	Theoretical velocity.	Coefficient.	Direction and force of wind.
		Area.		Mean depth.	Width.	Scour or fill.						
		Water.	Below datum.									
1884.	Feet.	Sq. ft.	Sq. ft.	Feet.	Feet.	Sq. ft.	Feet.	Cu. ft.				
December 12	7.82	96 848	209 073	31.1	3 114	— 176	2.418	234 212	0.00003732	3.41	71.0	W., strong.
13	7.42	94 694	208 202	30.4	3 111	— 871	2.338	221 390	0.00003746	3.37	69.3	N. E., light.
14	7.50								0.00003784			
15	8.84	99 689	208 684	32.0	3 115	+ 482	2.438	242 997	0.00004100	3.62	67.3	Calm.
16	10.94	107 009	209 306	33.1	3 229	+ 622	2.676	286 283	0.00004453	3.75	69.7	S. E., light.
17	12.76	112 401	208 851	34.5	3 255	— 455	2.853	320 685	0.00004632	4.00	71.4	N. W., strong.
18*	15.01	116 979	306 087	35.8	3 268	—2 764	2.883	337 316	0.00004688	4.10	70.5	Do.
19	16.57	124 801	208 810	38.0	3 282	+2 723	2.925	365 023	0.00004806	4.27	68.4	N., light.
20	17.58	127 399	208 089	38.8	3 286	— 721	3.059	389 672	0.00004806	4.32	70.8	S. S., light.
21	18.10								0.00004782			
22	18.41	131 855	209 820	40.0	3 296	+1 731	3.144	414 584	0.00004806	4.38	71.7	N., strong.
23	18.41	128 313	206 283	38.9	3 296	—3 537	3.060	392 724	0.00004829	4.33	70.6	N., light.
24†	18.14	128 980	207 834	39.1	3 295	+1 551	3.076	396 752	0.00004759	4.32	71.2	N., strong.
25	17.85								0.00004749			
26	16.80	123 438	206 689	37.6	3 281	—1 145	2.957	365 000	0.00004658	4.18	70.7	Calm.
27	15.85								0.00004477			
28	15.35								0.00004571			
29	14.98	117 421	206 633	35.9	3 270	— 56	2.895	339 965	0.00004905	4.19	68.9	E., strong.
30	16.85	121 488	204 574	37.0	3 281	—2 059	3.093	375 724	0.00004759	4.20	73.7	N., strong.
31	20.28	131 365	203 150	39.8	3 299	—1 424	3 486	457 871	0.00005168	4.53	76.9	N. W., strong.
1885.												
January 1†	23.33	142 439	204 104	43.1	3 305	+ 954	3.913	557 296	0.00005325	4.79	81.6	N. W., very strong.
2	26.13	154 537	206 901	46.6	3 313	+2 797	4.123	637 141	0.00005371	5.00	82.4	N., light.
3	28.71	162 507	206 285	48.8	3 328	— 616	4.428	719 521	0.00005433	5.15	86.0	Calm.
4	31.10								0.00005528			
5	32.90								0.00005499			
6	34.55								0.00005645			
7	35.55	184 498	205 322	54.9	3 359	— 963	4.837	892 383	0.00005688	5.59	86.6	S., light.
8	36.30								0.00005726			

	9	36.85	194 569	211 039	57.7	3 370	+5 717	4.982	969 430	0.00005740	5.75	86.6	N., light.
	10	37.30	201 608	216 559	59.8	3 370	+5 520	4.863	980 502	0.00005773	5.87	82.8	E., light.
	11	37.60								0.00005670			
	12	38.10	207 121	219 374	61.4	3 371	+2 815	4.907	1 016 372	0.00005679	5.90	83.1	N., light.
	13	38.49	202 053	212 990	59.9	3 371	−6 384	4.798	969 431	0.00005735	5.86	81.9	N., strong.
	14	39.03	202 334	211 448	60.0	3 371	−1 542	5.167	1 045 446	0.00005773	5.89	87.8	N. E., light.
	15	39.80								0.00005858			
	16†	40.28	207 854	212 748	61.7	3 371	+1 300	5.374	1 117 022	0.00005867	6.02	89.3	N. W., strong.
	17	40 61	207 350	211 131	61 5	3 372	−1 617	5 152	1 068 270	0.00005867	6.01	85.8	W., strong.
	18	40.95								0.00005927			
	19	41.04	207 404	209 734	61.5	3 374	−1 397	5.307	1 100 708	0.00005923	6.04	87.0	N., light.
	20	41.20	209 210	211 000	62.6	3 375	+1 266	5.114	1 069 814	0.00005914	6.08	84.1	N. W., light.
	21	41.30								0.00005923			
	22	41.35	207 877	209 161	61.6	3 376	−1 839	5.158	1 072 266	0.00005891	6.02	85.6	N., light.
	23	41.48	211 451	212 296	62.6	3 377	+3 135	5.165	1 092 097	0.00005891	6.07	85.1	Calm.
	24	41.50								0.00005900			
	25	41.50								0.00005853			
	26	41.50								0.00005867			
	27	41.60								0.00005914			
	28	41.70								0.00005952			
	29	41.70								0.00005914			
	30	41.69	218 608	218 748	64.7	3 380	+6 452			0.00005891	6.17		
	31	41.73	217 778	217 778	64.4	3 380	— 970	4.888	1 064 462	0.00005867	6.15	79.5	W., light.
February	1	41.70								0.00005867			
	2	41.61	216 234	216 640	64.0	3 380	−1 138	4 979	1 076 725	0.00005881	6.13	81.1	S., light.
	3	41.34	216 288	217 606	64.0	3 380	+ 966	4.853	1 049 734	0.00005797	6.09	79.7	Do.
	4	40.79	217 134	220 308	64.4	3 374	+2 702	4.727	1 026 309	0.00005773	6.10	77.5	S., strong.
	5	39.90	215 970	222 147	64.1	3 371	+1 839	4.541	980 803	0.00005561	5.97	76.1	W., very strong.
	6	38.72	212 233	222 393	63.0	3 371	+ 246	4.382	929 904	0.00005628	5.90	74.3	S., light,
	7	37.46	205 316	219 729	60.9	3 371	−2 664	4.229	868 374	0.00005410	5.74	73.6	S., strong.
	8	36.30								0.00005324			
	9	35.14	193 968	216 189	57.7	3 364	−3 540	4.058	787,120	0.00005301	5.53	73.4	W., strong.
	10	34.15								0.00005277			
	11	33.59	188 421	215 837	56.2	3 356	— 352	3.944	743 225	0.00005395	5.51	71.6	S., light.
	12	33.19	184 158	212 912	54.9	3 354	−2 925	3.966	730 365	0.00005386	5.44	72.9	N.
	13	32.93	184 790	214 420	55.1	3 354	+1 508	4 074	752 836	0.00005409	5.46	74.6	N., light.
	14	32.70	184 080	214 484	54.9	3 354	+ 64	4.171	767 829	0.00005409	5.45	76.9	Calm.
	15	32.55								0.00005442			
	16	32.25	179 871	211 785	53.6	3 353	−2 699	4.078	733 525	0.00005456	5.41	75.4	W., light.
	17	32.18	181 758	213 908	54.2	3 353	+2 123	4.178	759 442	0.00005433	5.43	77.0	S., light.
	18	32.25	179 674	211 588	53.6	3 353	−2 320	4.090	734 787	0.00005442	5.40	75.7	W., strong.
	19	32.30								0.00005451			
	20	32.20	180 956	213 039	54.0	3 353	+1 451	4.188	757 822	0.00005433	5.42	77.3	N., strong.

* Reject. † High waves.

TABLE No. IV.—Discharge Observations at Arkansas City, Ark.—(*Continued*).

Date.	Gauge reading.	Dimensions of Cross-Section of Discharge.					Mean velocity per second.	Discharge per second.	Slope, Arkansas City-Greenville.	Theoretical velocity.	Coefficent.	Direction and force of wind.
		Area.		Mean depth.	Width.	Scour or fill.						
		Water.	Below datum.									
1895.	Feet.	Sq. ft.	Sq. ft.	Feet	Feet.	Sq. ft.	Feet.	Cu. ft.				
February 21	31.97	180 207	213 064	53.7	3 353	+ 25	4.028	725 963	0.00005442	5.41	74.5	N. E., light
22	31.35								0 00005334			
23	30.52	175 006	212 694	52.3	3 344	− 370	3.822	668 902	0.00005301	5.27	72.6	N. E., strong.
24	29.64	168 823	209 499	50.5	3 342	−3 195	3.707	625 866	0.00005254	5.15	72.0	N. W., light.
25	28.45	162 064	206 738	48.6	3 338	−2 761	.....	.....	0.00005221	5.04	.....	Calm.
26	27.05								0.00005004			
27	25.39	152 179	207 050	45.7	3 331	+ 312	3.364	510 424	0.00005004	4.78	70.1	S , light.
28	24.01	146 272	205 744	43.9	3 328	−1 306	3.311	484 270	0.00004876	4.63	70.6	E., light.
March 1	22.80								0.00004867			
2	22.09	144 366	210 146	43.7	3 302	+4 402	3 255	469 891	0.00004914	4.63	70.1	Calm.
3	21.81	144 518	211 225	43.8	3 301	−1 079	3.166	457 584	0.00004970	4.67	67.9	Do.
4	21.77	145 777	212 617	44.2	3 301	+1 392	3.266	476 135	0.00005027	4.71	69.3	N. E., strong.
5	21.85	144 162	210 736	43.7	3 301	−1 881	3.182	458 682	0.00005032	4.69	67.9	N., strong.
6	22.15	145 437	211 013	44.1	3 301	+ 277	3.237	470 825	0.00005065	4.73	68.5	S. W., light.
7	22.62	146 945	210 962	44.5	3 302	− 51	3.340	490 803	0.00005159	4.79	69.7	N., strong.
8	23.15								0.00005207			
9	23.83	152 241	212 298	45.8	3 323	+1 336	3.309	503 789	0.00005278	4.92	67.3	S. E., light.
10	24 53	154 370	212 111	46.4	3 330	− 187	3,460	534 116	0.00005362	4.99	69.4	S. W., strong.
11	25.48	155 650	210 222	46.7	3 332	−1 889	3.486	542 651	0.00005381	5.02	69.5	S. W., light.
12	26.70	158 835	209 340	47.6	3 337	− 882	3,576	567 990	0.00005490	5.11	70.0	S., light.
13	28.08	164 755	210 648	49.3	3 340	+1 308	3.827	630 445	0.00005561	5.24	73.1	W., light.
14	29.26	168 436	210 385	50.3	3 346	− 263	3.747	631 086	0.00005584	5.30	70.7	S., light.
15	30.20								0.00005575			
16	30.87	174 438	210 998	52.0	3 353	+ 613	4.002	698 138	0.00005551	5.37	74.5	S., light.
17	31.37	174 567	209 454	52.0	3 355	−1 544	4.107	716 933	0.00005537	5.36	76.5	N., strong.
18	32.02	180 981	213 699	53.9	3 359	+4 245	4.100	741 980	0.00005593	5.49	74.7	N. W., light.
19	32.71	178 824	209 230	53.2	3 362	−4 469	4.287	766 661	0.00005622	5.47	78.4	W., strong.
20	33.41	180 488	208 543	53.6	3 364	− 687	4.316	779 062	0.00005688	5.52	78.2	N., light.
21	34.01	184 859	210 902	54.9	3 367	+2 359	4.186	773 810	0.00005679	5.58	75.0	

	22	34.40								0.00005608			
	23	34.75	187 655	211 209	55.7	3 369	+ 307	4.202	788 488	0.00005584	5.58	75.4	N., light.
	24	34.83	192 356	215 643	57.1	3 370	+4 437	4.165	801 188	0.00005608	5.66	73.6	N. E., light.
	25	34.86	188 825	212 011	56.0	3 370	−3 632	4 193	791 786	0.00005670	5.63	74.4	Do.
	26	34.75	189 316	212 873	56.2	3 370	+ 862	4.234	801 335	0.00005617	5.62	75.4	Calm.
	27	34.55	188 810	213 039	56.0	3 369	+ 166	4.181	789 380	0.00005546	5.57	75.0	S., light.
	28	34.28	188 165	213 290	55.9	3 365	+ 251	4.352	818 810	0.00005504	5.55	78.5	N. W., strong.
	29	33.90								0.00005499			
	30	33.30	185 640	214 058	55.2	3 362	+ 768	4.002	742 849	0.00005433	5.48	73.1	S., light.
	31	32.60	181 149	211 908	53.9	3 358	−2 150	3.946	714 826	0.00005395	5.39	73.2	Calm.
April	1	31.75	178 475	212 078	53.2	3 354	+ 170	3.851	687 259	0.00005419	5.37	71.7	S. E., light.
	2	30.80	173 935	210 714	51.9	3 350	−1 364	3.844	668 617	0.00005301	5.25	73.3	S., strong.
	3	29.90	167 984	207 790	50.1	3 351	−2 924	3.896	654 455	0.00005240	5.12	76.1	N. W., strong.
	4	28.86	165 699	208 986	49.5	3 345	+1 196	3.729	617 823	0.00005254	5.10	73.1	N., light.
	5	28.00								0.00005183			
	6	27.34	159 934	208 303	47.9	3 340	− 683	3.512	561 739	0.00005159	4.97	70.7	S., light.
	7	26.92	157 807	207 579	47.3	3 339	− 724	3.553	560 652	0.00005159	4.94	71.9	S. E., light.
	8	26.77	157 628	207 908	47.2	3 343	+ 329	3.544	558 663	0.00005254	4.98	71.2	E., strong.
	9	27.06	157 554	206 864	47.1	3 343	−1 044	3.601	567 279	0.00005301	5.00	72.1	S. E., light.

NOTE.—Most of the computations of theoretical velocities, etc., in this paper were made with the slide rule. Hence there may frequently be errors in the final figures.

according to the formula $v = 100\sqrt{rs}$, and the coefficients required to bring the expression $\sqrt{rs}$ to the velocity as observed.

Curve of Coefficients.—The principal object of the investigation is to arrive at the relation between discharge and gauge height. It is evident from a casual inspection of the table that the coefficients, on the whole, increase as the river rises and diminish as it falls. To develop the law of this change, take the gauge as an argument and

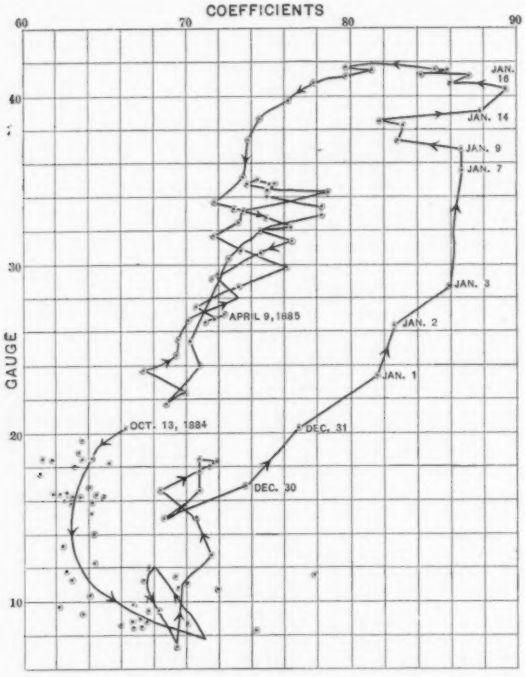


FIG. 20.

plot the coefficients as functions, as in Fig. 20. It will be seen that they do not by any means follow a regular law of progression with the gauge, but that the curve which they describe is an irregular oval, the greater values of the coefficients corresponding, on the whole, to the ascending branch, representing a rising river, and the less values to a falling river. This general law is subject, however, to some

remarkable exceptions, there being violent departures from the curve, sometimes followed by as sudden returns. If the table of data is examined, it will be found that these, for the most part, as might have been expected, are contemporaneous with changes of area in the opposite direction. A further examination will reveal the fact that the two branches of the curve themselves, the ascending and the descending, also represent less and greater areas respectively.

Arkansas City has a narrow and deep section; in short, the river there is a pool, and in accordance with the general law the cross-section undergoes enlargement by scour with a rise, and contraction by fill with a fall. Now, if the controlling section of a river, whatever that be, remained unaltered, every change in the area at Arkansas City would be exactly compensated by an inverse change of velocity. The discharge, indeed, would not always be the same for the same gauge height, for the velocity would still be affected by the changes of slope due to rises and falls; but the coefficients, which take cognizance of the changes of slope, would follow a regular law, subject to small perturbations.

On the other hand, a persistence of datum area at any station, while the adjacent reaches, above or below or both, are undergoing mutation, will produce a change of velocity at the station, even though the general slope and mean depth remain the same; or if the latter elements also vary, as they usually do, then the change of velocity will be out of all proportion to the variation in those elements; or, in other words, the coefficient will show a marked departure from its ordinary rate of increase or decrease. The same effect will follow in a less degree if the areas do not remain absolutely persistent, but fail to respond at once to the altered conditions elsewhere. A very striking exemplification of this statement is afforded by the high-water discharge of 1890 at Wilson's Point, of which a plotting is given in Fig. 12, previously referred to. It will be seen that the coefficient varied, from January 28th to March 13th, from 146 to.97, the river steadily rising. The movement of the coefficient, on the whole, was tolerably regular. Observing now the datum areas, it will be found that they increased steadily from about 159 000 to 193 000 during the same time, and remained at about that figure till the close of the observations. The inference is irresistible that here is the true explanation of this phenomenon; and it shows the futility of attempting to apply formulas

based on general conditions to the particular circumstances existing at any one place, without a knowledge of those circumstances and a careful scrutiny of the disturbing causes which they introduce.

It is not necessary to assume any alteration in the controlling section to account for the increased velocity at the discharge station. The latter, as at Arkansas City, and as is generally the case, is a narrow section, much less than the average of the river, and the rapid rise had poured down a flood, which increased faster than the contracted section at Wilson's Point could accommodate it at the usual velocity. Hence the latter had to be increased, which event happened, no doubt, by steepening the local slope, though the movement was attended by so slight a disturbance of the water level that it made little or no impression on the general slope. The velocity of 7 ft. per second or more that was attained rapidly produced a scour, and, as the area enlarged, the velocity fell by degrees to the normal.

An equally significant example of the fallacious indications afforded by strictly local conditions is presented by the Helena observations of 1884-85. At this point there were two sections, one for high and the other for low water, about 0.75 mile apart. On two days measurements were taken at both. The general slope, of course, was the same for both sections. Yet at the upper (low water) station, on a given date, the mean depth was 19.7, the velocity 3.06, and the coefficient 71, while at the lower the mean depth was 14.1, the velocity 3.82, and the coefficient 105. On a subsequent date the velocities were 2.85 and 3.59, the coefficients 71 and 95. On the first day the discharges measured at the two sections differed by 25 000 cu. ft.; on the second they were almost identical in value.

Formulas Give Too Low Velocities at Low Stages.—It may here be remarked that the formulas all give too low values for the velocity at low stages, and consequently too high values for the coefficients, and those formulas which, like Weisbach's and Kutter's, give especial prominence to the depth, err particularly in this regard. The reason for this error is that they take the general mean depth of the section, obtained by dividing the area by the wetted perimeter, instead of dividing the section into partial areas and applying the formula to each, an error identical in principle with that mentioned on page 54, whereby the mean velocity is confounded with the mean of the mean velocities in vertical planes. The amount of error thus introduced

will vary with the shape of the section and the stage of water, being greatest when the section is composed of a deep and a shallow part, and when the latter is first submerged. In sections of uniform depth it is nothing Generally it is greatest at low stages, the variations of depth being relatively less when the river attains its height. In the present instance, at the lowest stage, 7.42 ft. on the gauge, the error is about 10%; at 20 ft. it is about 8%; at the bank-full stage it is about 4 per cent.

Fill at Low Stage.—An inspection of Table IV will show that the fill progresses very steadily at low stages, the area decreasing from 230 000 to 203 000. It may be a matter of surprise that at so low a stage, when the river has so little transporting power, the movement of material should be so considerable and so persistent, when at much higher velocities, as from December 30th to January 23d, it should be comparatively insignificant. It should be remarked, however, that the material which is deposited on the bottom is not brought from afar, but is, in all probability, derived from the bar immediately above, and that the velocity prevailing on that bar is abundantly sufficient to dislodge and transport considerable quantities of sediment, the deposition of which is favored by the low velocity in the pool. It is probable also that a good deal of matter is rolled along the bottom.

The relation between transporting power and velocity is not definitely known, and there is some diversity in the views of engineers on the subject. It is generally admitted, however, that there is a connection between the two, and that an increase of velocity is attended by a scour, and a diminution of velocity by a deposit of the matter already in suspension.

Supposition of Uniform Velocity.—Now suppose a stream, straight in plan and of erodable bed, to be actuated by a current which is perfectly uniform. If the water enter the head of this stream already loaded with all that it can carry at that velocity, it will take up no more, but will transport its burden without accession or loss to the sea. If the course have bends in it, the force of the current will be exerted mechanically against the banks presented at abrupt angles, and an additional load will be temporarily taken up to be deposited presently at the reversion points of the bends. The general result will be the same, that a load will be delivered at the mouth equal in quantity to that carried at the head of the alluvial basin. This, of course, on

the supposition that the material carried is homogeneous with that of the bed and banks.

There will still be a quantity of matter rolled along by the current. According to the received ideas, there is always velocity enough prevailing in the Mississippi to move the loose and finely divided material which lines its channel. From the well-worn tables of Dubuat it would appear that a mean velocity of less than 1 ft. per second will move sand and even coarser substances. Both Mr. Powless and Mr. Hider found the movements of sand waves to proceed at low water as well as at high. The movements, however, were more violent and the mass of material moved was greater as the velocity increased.

Scour or Fill Depends on Differences of Velocity.—It will therefore be seen that the scour or fill at any station does not depend on the absolute velocity at that place, but on the difference of velocity between it and the adjacent station. If the velocity were the same, no matter how great, the same quantity would leave the section as entered it. While, therefore, there might be a very large movement of material through the section, measurements would not show any permanent changes in its dimensions. There might be considerable daily disturbances, due to the passage of sand waves.

The alternation of wide and narrow places is very well exemplified at Arkansas City. About 3 miles above the discharge section, which is about 3 400 ft. wide, is a wide and shallow stretch about 7 750 ft. broad. According to the soundings of 1882, at a stage of 42.4 on the Arkansas City gauge, the area of this section was about 233 000 sq. ft., while that of the narrow section just below the town was 203 700. If the areas are assumed to remain unaltered by fill or scour, there will be found at 35 ft., areas of 178 300 and 178 400; at 30 ft., 139 700 and 161 400; at 20 ft., 119 800 and 128 500; at 7.4 ft. (low water of 1884–85) 51 600 and 86 500. Fig. 21 gives plots of the two sections. It will be observed that at the extremely low stage, the wide place has contracted its breadth to hardly more than half of its former proportions, and has become almost a pool itself. These conditions, of course, do not continue, for the small cross-section enlarges and the great one contracts, at either stage, by the action of the river.

There is a time during the rise and also during the fall of every flood when the velocities in the deep and in the shallow places approach equality. At such times there should be little general change in the dimen-

sions of the bed. At Arkansas City, in 1882, assuming the bed to remain as it was when the soundings were made for the general map, at a stage of about 35 ft. on the Arkansas City gauge, the area in the two sections was about the same, namely, about 178 400 sq. ft., and the velocity about 4.66. Under such circumstances the changes in the dimensions of the sections should be principally from material rolled along the bottom.

The tendency, then, at each stage, is toward equalization of the areas of the wide and narrow sections. As soon as the area, either on

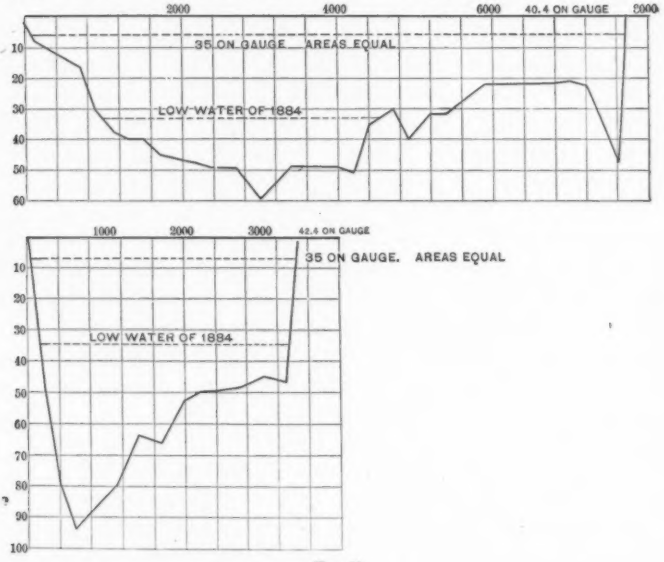


FIG. 21.

bar or in pool, becomes less than the one adjacent, its velocity relative to the latter is augmented, and scour takes place, accompanied by fill in the place where the current is slackened. This deduction is borne out by the recorded data. A gradual fill in the discharge section is seen at stages below about 34 or 35 ft., and this irrespective of rise or fall. There is every reason to suppose that the material for this fill is derived from scour of the bar above. At stages above 35 a tendency to scour is observed in the discharge section; not regular, however, but fitful. At the extreme high water of 1885, the areas of

the two sections, according to the survey of 1882, should be respectively about 227 600 and 201 300. The mean of these is about 214 500. To this limit, then, both areas would tend. Toward the climax of the rise the cross-sections are found to vary from about 207 900 to 218 600, the areas being maintained for several days above the mean, either from the passage of sand waves or other accidental circumstances, or more probably because the soundings of 1884–85, made at intervals of 50 ft., gave larger areas than those of 1882, at intervals of 250 or 300.

At a stage of 50 ft., the two sections should have areas of about 292 000 and 230 000; mean, 261 000. In 1893, the river actually reached this height. The datum area at the discharge section, which had been about 230 000 at a stage of 35, increased to 244 000 at 46, and then rapidly scoured out to 261 000 at 50, where it remained till the close of the observations.

Increase of Coefficient with Gauge Height.—In examining the curve of coefficients, they are seen to increase with the height of the stage, and this is generally understood to be the law of the progression. All formulas which do not make c a constant make it increase with either the velocity or the mean depth, or both. Is this true? And if so, why is it true?

First.—The roughness of the banks and bed cuts a somewhat greater figure at low water than at high. At a mean depth of 60 or 70 ft., it makes comparatively little difference whether the sides of the channel be rough or smooth. At 20 ft., its effect may be quite perceptible, though still small. According to Kutter's formula, at depths of 64 and 25 ft., there is hardly more than 1% difference in the coefficient of velocity for a considerable change in the coefficient of roughness.

Second.—At some stations, such as Paducah and Helena, there is an increased ratio between mean and bottom velocities at low water. If it be true, according to the supposition of M. Dupuit, previously quoted, that the velocity of the formula is the bottom and not the mean velocity, then the more nearly these quantities approximate, the more nearly will the formula give the mean velocity. Now the mean velocity is always the greater. To obtain it from the velocity of the formula a coefficient will be required, and this coefficient will be greater as the disparity between the mean and bottom velocities is more considerable. At Paducah the ratio between the maximum and

bottom velocities is, at 60 ft. depth, 0.47; at 10 and below, 0.76. At Helena, the same quantities are 0.58 and 0.71. The ratio between mean and bottom velocities is probably, for the mean of the two stations, about 0.60 at high water and 0.80 at low.

The observations at the other stations prove nothing. At Columbus there were only twelve measurements taken below 50 ft.; at Hays's Landing no bottom velocities were taken at all, and at Red River the record is extremely scanty and unsatisfactory.

Third.—The disparity between the general and local slopes is greatest at low water. For obvious reasons the discharges are always taken at narrow and deep sections. These become almost ponds at low water, with a great area and a low velocity, accompanied, of course, by a flat local slope. The computed or theoretical velocity, obtained from the general slope and the local mean depth, is, therefore, at such times excessive, and to reduce it to the observed velocity the coefficient must be made very small. On the other hand, at high water, the local slope approximates to the general, as has been explained. In deep sections the velocity at high water is often three or four times as great as at low, while in the wider places it is probably less than twice as much. On the bar above Arkansas City, in 1884–85, at high water, the mean velocity was probably about 4.8 ft., at low water about 3.5 ft. At the discharge section it ranged from 5.3 to 2.3. At Helena, in 1884, the velocity was 3.59 on the wide section when it was 2.85 on the narrow, at extreme low water.

On the other hand, at small depths in irregular sections, the coefficient may exhibit a considerable increase, for the reason formerly explained, namely, from the computation by the whole section instead of by partial sections.

Of these reasons the third has far more influence than all the others put together. It will therefore appear that the increase in value of the coefficient with gauge height is not a necessary incident, but is principally dependent on the shape of the section. Of this, abundant confirmation will be given.

Explanation of the Irregularities of the Curve of Coefficients.—It is now possible to explain all the vagaries of the curve of coefficients. The greater values on a rising river are due to the smallness of the areas, as in the case of Wilson's Point in 1890. The area cannot increase by scour until the area of the bar above exceeds it, and, hence, there

is a disproportionate augmentation of velocity above what would have been expected from theory.

The smaller values after the crest of the rise has passed are owing to the fact that the areas which have lately scoured out have not yet returned to their normal proportions, and hence the velocities are less than they should be. The areas cannot experience much fill until the cross-section over the bar has been materially reduced, a scour established there and a considerable slackening in the pool below to cause a deposit.

The remarkable perturbations that are seen in the upward branch of the curve at 36.5 to 39 ft. are due principally to the violent fluctuations in the area, to which reference has already been made, whereby a scour of 14 000 sq. ft. occurred in three days, followed by a fill of 8 000 sq. ft.

The velocity on January 13th was undoubtedly a good deal too small, as is evident from the fact that the area decreased at the same time, giving a discharge probably 60 000 less than it should have been. The velocity on January 16th is probably considerably in excess of the truth, as the area shows very little change, while the discharge is given as about 60 000 more than the mean of the preceding and following days, and greater than on any other day of the season. The notes say: "High waves."

In the lapse of time between January 23d and 30th, when no observations were taken, owing to the loss of the meter, a sudden and rapid scour began, which continued until February 6th, when it was succeeded by a fill. The record shows a sudden decrease in the velocity and in the coefficient, corresponding to the increase of area. A new meter came into use at this time, and, unfortunately, the observations before and after January 31st cannot be compared with any assurance of accuracy. A meter may be consistent with itself, but, unless with the best instruments and the most careful rating and manipulation, the results given by different meters must be expected to present some discrepancies. Judging from the record subsequent to February 12th, when still another instrument was substituted, the meter used from January 31st to February 12th gave velocities too low by 0.1 ft. per second or more.

Supposition of an Invariable Datum Area.—While the movements of the curve of coefficients have been accounted for, they have not been

quantitatively estimated, nor has any means been indicated whereby the coefficient at any stage, under any circumstances, may be ascertained.

In order to get rid of the perpetually recurring changes of area, the datum section at Arkansas City may be assumed to have been invariable for a certain period, say, during the rise in January, the water area being subject only to such changes as were produced by rise and fall. In such a case it is evident that the discharges would pass with certain velocities which might be very different from those which actually prevailed. Those velocities are easily ascertained, however, by computing the water area which would have prevailed if the section had been unaltered, and dividing the discharge by it. The area is easily computed by taking the difference between the proposed datum area and the actual datum area on any given date, and adding it to or subtracting it from the given water area. The standard area will be assumed at about that which prevailed during the greater part of the rise, namely, 212 000 sq. ft. The velocities which would have existed on that supposition are as follows:

Date.	Gauge.	Assumed area. Square feet.	Assumed velocity. Feet per second.	Assumed mean depth. Feet.	Theoretical velocity. Feet per second.	Coefficient.
December 31....	20.28	140 215	3.27	42.5	4.69	69.8
January 1....	23.33	150 335	3.78	45.5	4.92	76.9
2....	26.13	159 636	3.99	48.2	5.09	81.7
3....	28.71	168 222	4.28	50.6	5.74	78.4
7....	35.55	191 176	4.77	56.7	5.68	83.0
9....	36.85	195 530	4.96	58.0	5.76	86.1
10....	37.30	197 049	4.98	58.5	5.81	86.0
12....	38.10	199 751	5.09	59.3	5.81	87.6
13....	38.49	201 063	4.82*	59.6	5.85	82.5
14....	39.03	202 886	5.15	60.2	5.90	87.3
16....	40.28	207 106	5.39*	61.5	6.01	89.7
17....	40.61	208 219	5.18	61.8	6.02	86.1
19....	41.04	209 670	5.25	62.1	6.07	86.5
20....	41.20	210 210	5.10	62.5	6.07	84.0
22....	41.35	210 716	5.10	62.6	6.07	84.0
23....	41.48	211 155	5.18	62.7	6.08	85.3
31....	41.73	212 000	5.02	63.0	6.08	82.4

If these velocities are plotted to the gauge heights, a pretty regular line will result, considering the crudeness of the data, approximately straight, except at the top of the rise, when it begins to break a little (see Fig. 22). The velocities marked * are considered untrustworthy, on account of the large variations of discharge already alluded to. The observations from January 1st to 10th are not quite consistent

with one another, for they make a greater variation in the discharge at low stages than at high, the rise being still in active progress, which is never the case. The discharges for those dates should be about 531 000, 607 000, 684 000, according to the usual rate of progression of the discharge. If these figures are used, the assumed velocities for those dates will be as represented by a △ in the plot.

Slackening of Velocity at Crest of Rise.—In endeavoring to find a law for the upward progression of the velocity or of the coefficient, the data obtained near the top of the flood should not be used, as a

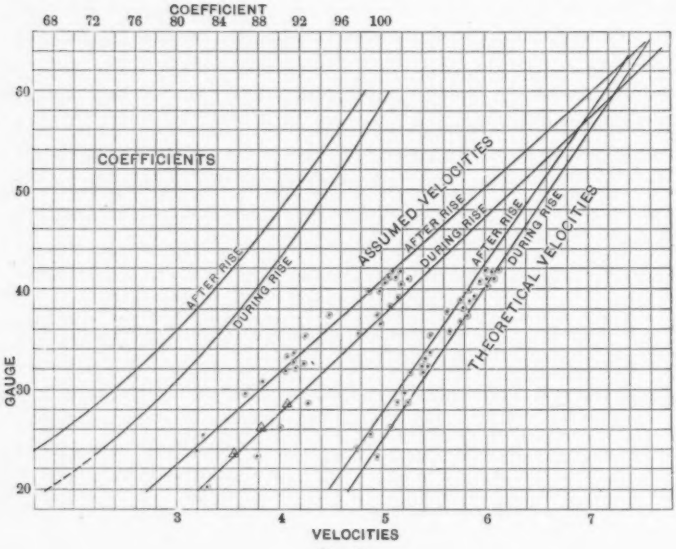


FIG. 22.

change in the condition of the river has already begun. It is a well-known fact, remarked long ago by Humphreys and Abbot, that a slackening generally occurs during the last foot or so of a rise, and that a less discharge is passed at the climax of the upward movement than at a stage a little lower. This fact is attributable to two causes.

First.—The slope begins to flatten at this time. As the rise falls off, the rate of progression of the gauges is diminished more rapidly above than below. It is rising at White River only 0.1 ft. while it is still rising 0.2 ft. at Arkansas City. Therefore the velocity also begins to decrease

even while the river is still rising. A cursory study of the slope curves on Figs. 16 and 17 will make this very clear.

Second.—There is a change occurring in the conformation of the river-bed. While the rise was in active progress, the bar was building and the pool was scouring. The scour did not begin in the pool, however, until the velocity in the latter surpassed that of the water on the bar, and the two processes continued until the two areas were equalized.

During this time, at all stages, the bars have acted as dams. If it had not been for them, the velocity in the pools, from the great depth prevailing in them, would have become excessive. The profile of the river has been somewhat as shown in Fig. 23. It is evident that the quantity of water which can be admitted to the pools is regulated by the height of the bars, and the conditions prevailing in the pools can only partially modify their effect. The velocity of the water at the wide sections is always greater than the theoretical velocity, because the local slope prevailing on the bar is in excess of the general slope. If the sections remained unaltered, a time would arrive when the general slope would be the greater. The rise produces a much greater proportional increase of mean depth on them than in the pools. At Arkansas City, for instance, a rise of 5 ft. above the gauge of 42.4 produces an increase of mean depth in the shallow section from 30 to 35, or one-sixth. In the deep section, the increase is from 60 to 65, or one-twelfth. Hence the velocity increases faster on the bars than in the pools, and at a stage of 55 ft. it would finally surpass the theoretical velocity. This process is modified by the alterations which take place in the bed. Not only is the pool scouring, but the bar is building; each pool furnishing the material for the accessions to the bar below it. While the upward progress of the river is tolerably regular and rapid, the daily increase of depth over the bar by virtue of the rise is more than sufficient to keep pace with the fill. When there is a considerable slackening in the rate of rise, however, as happens near the climax of every flood, the fill may exceed the increase in height of the water, and the bars may become veritable dams, increasing the depth in the pools while the discharge is the same, or maintain-

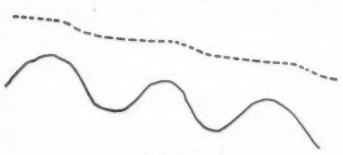


FIG. 23.

ing it when the supply of water is continually diminishing. For instance, from January 23d to January 31st, the cross-section at Arkansas City increased 5 500 sq. ft., or 1.8 ft. in mean depth. This is probably deposited in the first wide section below, and is sufficient to raise it a foot or so, while the gauge height has increased only a fraction of a foot.

Line of Assumed Velocities.—If this movement be general on all the bars throughout the river, as is likely to be the case, it may produce very little effect on the slope as estimated from station to station. Hence, in spite of the maintenance of the slope and of the discharge, there may be a slackening of the velocity; indeed there always is, and this is prolonged into the fall which succeeds.

A line drawn through the assumed velocities shown in Fig. 22 is very nearly a straight line; in fact a straight line fits them better than any other.

On the same supposition of an invariable datum area of 212 000, with the slopes actually prevailing, the theoretical velocities may now be computed and plotted, that is, the products $100\sqrt{rs}$, and the coefficients required to bring them to the assumed velocities lately found. If the slope remained constant at all stages, the line of theoretical velocities would be a curve concave to the axis of ordinates. The slope, however, at this station increases with the gauge height; consequently, the theoretical velocities are found ranged, as nearly as may be, in a straight line, not parallel to the line of assumed velocities, but inclined to it and intersecting it as the gauge gets higher. By means of these two lines, the coefficients of any number of points may be determined, and they will be found to correspond pretty closely with those given on page 445. They form a curve, given also in Fig. 22. The lines can be extended to any stage, and the coefficient ascertained. By taking the slope from the slope curve, and the mean depth from the known addition to the gauge height, the velocity may be computed. The assumed cross-section may also be easily ascertained by adding the increase of area due to the given rise, to the datum area of 212 000. The discharge at the gauge height required may hence be easily ascertained. The velocity may be more easily found by simply prolonging the straight line representing the assumed velocities.

Care should be taken, however, in extending by analogy the conditions found at moderate stages to extreme high water. Usually this

is the best means known for arriving at a correct estimate of the conditions likely to prevail at such stages, the direct observations obtained at flood height being generally tainted with many elements of error, which it is very difficult to eliminate. All the checks that are available should be used, however.

Discharge Curves.

The ordinary method of arriving at the discharge at stages beyond those directly experienced is by prolonging the curve of discharges. Now a very superficial examination of any discharge curve will show a few leading points. The discharge, as a general rule, increases with the gauge height, and in an increasing ratio, which, if the bed of the river remained unchanged, would probably be tolerably regular. The rate would always be greater on a rise and less on a fall, from the known peculiarities of the slope. For this reason it has been considered proper, as has been already observed, in tracing a discharge curve, to select only times when the river was stationary, and thus eliminate this disturbing influence. It is found that this process gives a far more regular curve, yet there are still very great discrepancies observable; so persistent, and recurring with such regularity, that it is impossible they should be errors of observation. Even in the measurements of any single season, they are conspicuous, if the time of observation be at all prolonged; but they are most noticeable in comparing the data obtained from different years.

It was observed several years ago by Colonel Suter,* that in every series of discharge observations, no matter how scattered they may appear, it was possible to group them about a series of curves parallel to a certain standard curve. That is, the curve of one year or of one stage would not answer for another year or another stage unless its position were shifted up or down. This fact is strikingly visible in every series of observations, from the days of Humphreys and Abbot to the present time. The word "parallel" must not be understood always in a literal sense, for each curve is more or less affected by perturbations of one kind or another. The general effect is the same, as Colonel Suter remarks, as if the whole river were bodily raised or depressed.

* Paper on investigation of discharge measurements, in "Report of the Mississippi River Commission for 1891," p. 3419.

Discharge Curve of 1884–85 at Arkansas City.—Fig. 24 shows the principal features of the discharge curve of 1884–85 at Arkansas City. It will be seen that the curve consists of two principal branches, one representing the main rise of the season, in January, 1885, the other the fall which succeeded that rise and the movements subsequent thereto. It is evident that during the latter period almost the same relation prevailed between the successive values of the discharge as during the rise, the difference being that the discharges passed at

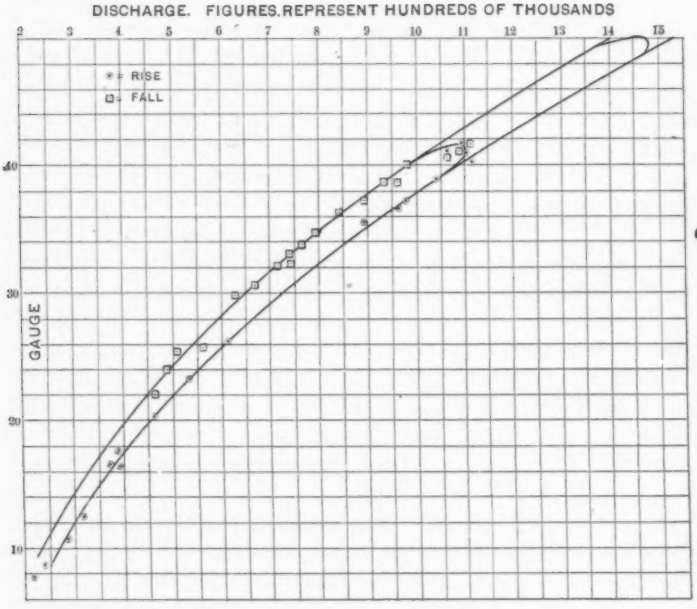


FIG. 24.

a gauge reading higher by a little more than 2 ft. after the crest of the rise had passed than during the progress of the swell. The two curves are very nearly parallel.

Now the discharge at any height is equal to the assumed velocity multiplied by the water area corresponding to that height, as computed from the standard datum area, supposed to be invariable.

At medium and high stages the area increases with great regularity. At 20.28 ft. the width at the water line was 3 299 ft. At 41.7

it was 3 380. The increase of area, therefore, for each 2 ft. of rise would be successively 6 608, 6 616, 6 624 ft., etc., so small a difference that if the values of the areas were plotted to the height as an argument to any ordinary scale, the line drawn through the points thus established would not be distinguishable from a straight line. But if the assumed velocities also range themselves in a straight line, the values of the products of the two should be represented by a series, the differences of which should run only to the second order. This in fact is the case. If the ascending and descending values of the discharge be arranged in two series, they will run very nearly as follows:

During Rise.				After Rise.		
Gauge.	Discharge	First difference.	Second difference.	Discharge.	First difference.	Second difference.
24.....	550 000	.....	.....	485 000	.....	.....
26.....	604 000	54 000	.....	535 000	50 000	.....
28.....	661 500	57 500	3 500	588 500	53 500	3 500
30.....	722 500	61 000	.....	645 000	57 000	.....
32.....	787 000	64 500	.....	705 500	60 500	.....
34.....	855 000	68 000	.....	769 000	64 000	.....
36.....	926 500	71 500	.....	836 500	67 500	.....
38.....	1 001 500	75 000	.....	907 000	71 000	.....
40.....	1 080 000	78 500	.....	981 500	74 500	.....
42.....	1 162 000	82 000	.....	1 059 000	78 000	.....
44.....	1 247 500	85 500	.....	1 140 500	81 500	.....
46.....	1 336 500	89 000	.....	1 225 000	85 000	.....
48.....	1 429 000	92 500	.....	1 313 500	88 500	.....
50.....	1 525 000	96 000	.....	1 405 000	92 000	.....

In this table the discharges are extended to a stage 10 ft. beyond that at which the observations terminated. If it is assumed that the behavior of the several quantities considered would be the same at higher stages as at the stage actually reached, then the velocities would follow the line pursued during the rise, until a short time previous to the culmination of the flood, when the velocities would break away, remain stationary for a while, the rise still progressing slowly, and then gradually slacken off; and during the fall they would approximately pursue a prolongation of the line which they followed during the same circumstances at the lower stage. If a line were drawn parallel to the two curves which have just been determined and passing through the point representing the discharge at the highest gauge reading attained, namely, 41.7, then the intersection of this curve with the horizontal line representing the height of 50 ft. would give the discharge for that reading. The mean of six observations

taken at the top of the flood gives a discharge of about 1 086 000. From either of the two series given on page 451 the increase of discharge corresponding to that value may be ascertained for an addition to the height of 8.3 ft. From the first series it is found by interpolation that a discharge of 1 086 000 corresponds to a height of 40.15. A height of 48.45 demands a discharge of 1 450 000. Then the increase of discharge is 365 000, and the discharge at the height of 50 ft., supposed to be at the stationary point, or at the turn of the curve, will be 1 450 000.

Changes of Plane.—The phenomenon of the shifting of the stage of the discharge is very appropriately called by Colonel Suter a "change of plane." It seems to be principally due to the changes which have been noted as occurring in the river-bed from stage to stage. The fill on the bars continues steadily, in harmony with the upward progress of the river, until, by the joint effort of the fill and of the contemporaneous scour in the pools, the areas, and hence the velocities, are brought to a state of equality. So during the progress of a fall, after the medium stage, or stage of equality of areas, has been passed (which at Arkansas City in 1884–85 seems to have been about 35 ft., varying, of course, with the changes of the bed), the wide places cut out and the pools fill, with a tendency toward the same result, equalization of cross-section and of velocity.

At high water, owing to the energy of the forces at work, the process goes on with great rapidity. At low water, the velocities are feeble and the changes from day to day comparatively small. If the season be short, the equilibrium of areas may never be reached. Probably it seldom is.

The medium stage would appear to be, on the whole, one of conservatism of existing conditions, whatever they may be. If this stage succeeds a considerable and sustained rise, it finds the bars high and the pools deep. Coming after a prolonged low stage, it finds the bars low and the cross-section in the pools small.

It has been shown that the bars are the controlling elements of the bed at all stages. A general elevation of the bars throughout the river, or throughout that portion in which the elements are considered, is equivalent to a general elevation of the water surface. For any one year, the oscillations of the datum area at the discharge section are generally pretty fair exponents of the changes from stage to stage on

the bars, the movements of the latter being in the inverse direction. For instance, in 1884–85, the continued decrease of area during the low stage indicated a progressive scour on the bars, and hence a "change of plane" in a downward direction; while at the top of the rise the rapid enlargement of the discharge section was significant of a fill in the wide places, and hence of a rise of the water plane.

From year to year, the phenomena have not necessarily the same significance. Thus, in 1889, with a datum cross-section of only 195 000, the discharge of 745 000 passed at a stage of 35 ft. In 1885, after the rise, with a datum area of about 212 000, the same discharge had passed at a stage of about 33, and during the rise, with an area of 206 000, at about 31. Hence it is probable that there are other changes of the bed, such as have been mentioned on pages 428 and 429, of a more permanent or perhaps of a periodic nature, which have not yet been carefully studied. Such changes may be supposed to proceed from a long continuance of high-water or low-water conditions; from an unusual predominance of floods from clear-water streams like the Ohio or upper Mississippi, or from silt-bearing tributaries like the Missouri and Arkansas, all which may very readily have an important effect upon the general bed, etc. Some importance may also be attributed to the maintenance of levees and the consequent confinement of floods, or, on the other hand, to the occurrence of crevasses and the deterioration of the bed which may naturally follow therefrom. The reclamation of tracts hitherto unprotected and the consequent exclusion of the action of basins as reservoirs may be included in the same category. These are very complicated matters, and require for their discussion much local knowledge and much careful study.

The respective parts played in the "change of plane" of the discharge by the several causes may be easily discriminated. In the case of 1884–85, the slopes corresponding to the two branches of the curve differ by an extreme quantity of about 0.000004. At a stage of 35 ft., this difference would produce a change in the velocity of about 0.1 ft. per second, or in the discharge of about 20 000 cu. ft. The actual difference was about 90 000. The change of area in the discharge section alone would not affect the discharge, but only the velocity. Hence the elevation of the bars was responsible for more than three-quarters of the discrepancy.

The change of plane is sometimes mostly local. On March 9th to

11th, 1893, a discharge estimated in the mean at about 1 050 000 passed Arkansas City at a stage of 43.1, while in 1885 the same discharge passed between 39 and 40. In this instance the change of plane was confined principally to Arkansas City, the gauge at that point being much higher, relatively, to White River and Greenville than in 1885. At the height of the rise in the latter year, the following figures show the readings of the gauge at White River, Arkansas City, Greenville, Lake Providence and Vicksburg, compared with March 9th to 11th, 1893.

	1885.	1893.
White River	43.60	44.3
Arkansas City	41.70	43.2
Greenville	37.60	37.7
Lake Providence	34.75	35.8
Vicksburg	42.40	42.4

Thus, it will appear that there were slight changes of plane at White River and Lake Providence, a considerable one at Arkansas City, and none at all at Greenville or Vicksburg.

High-Water Conditions Should Repeat Themselves.—If the high water be of long duration, the slope should be brought to very nearly the same value from year to year, the depths of water on the bars should be nearly the same, and, in short, the conditions should repeat themselves, except in so far as they are affected by permanent changes of the bed. Suppose the bed of the river to be as represented by the line *C D E F* in Fig. 25, and suppose the water surface

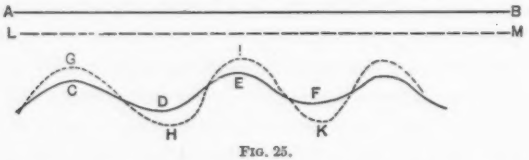


FIG. 25.

at the height of the flood to be as represented by *A B*. The area on the bars being much greater than in the pools, the velocity will be slackened and the bars will build. In the pools the velocity will be increased and the bottom will be depressed. The bed will then assume the shape *G H I K*. Now this reasoning is altogether independent of the exact shape of the original bed *C D E F*. It may be as it is figured in the sketch, or it may be already in a position approximating

that shown by *G H I K*. In the latter case there will be little change. If the river remain at flood height for a long time the result will be the same in the one case as in the other.

The end of the process will be reached when the areas at *G* and at *H* are equalized. But suppose the water level to occupy the position *L M*, lower than the normal, instead of *A B*; will not, then, the areas be equalized and the movement of material stop prematurely at a lower level than before? It appears that this should not occur unless the whole bed of the river undergoes a permanent change. The high-water slope is fixed or oscillates between limits. As long as the general bed of the river remains the same, and the movement of material is confined to the transference of silt from one locality to another nearly adjacent, the high-water slope will not be disturbed thereby, provided it has time to adjust itself. As the bars rise the water will rise also until it has attained its normal slope. If it be supposed possible that the process should extend further than this and the bars rise still more, the consequence would be that the level above the bar would be raised and the velocity over the bar increased, thus putting an end to the building process. In other words, as it has been expressed, "the slope would prove more inflexible than the bottom." *

The higher the flood, the more rapidly and certainly will the normal level be reached. If the flood be short and of moderate height, it is evident that the water plane at the extreme point reached may be above or below the normal, the conditions at medium stages having comparatively little tendency to produce change. The stage of water will then be largely controlled by the condition of the bed as the rise finds it. With the powerful forces prevailing at extreme flood, adjustment is speedily brought about. If a high water indefinitely prolonged is assumed, it appears a natural consequence that a certain height, slope and other conditions should be reached which would be reproduced under similar circumstances at other times, with the greater certainty and precision as the new conditions conformed to the old, and as the time was extended for their operation.

It would be desirable to bring this reasoning to the test of fact, but this cannot be rigorously done, for the true high-water slope of the Mississippi has never been ascertained, even for one year. It has always been disturbed by the same influences that have introduced so

* "Report of the Mississippi River Commission for 1884," p. 2551.

many complications into the movements of the discharge; escapes and returns of water, influence of tributaries, reclamation of lands, etc. It is also very much affected by duration of flood. Every one has remarked, however, the persistence with which high-water conditions repeat themselves. Any "river prophet," if given the stage at Cairo and its duration and the condition of the tributaries, will state within 0.5 ft. or less what height will be reached at Helena, Arkansas City, Greenville or Vicksburg from a comparison of previous experiences.

Extreme Discharges at Arkansas City.—The discharge at an extreme height has been measured at Arkansas City under very favorable conditions. The stage was 49.6, within 0.7 ft. of the highest ever reached there. The river was stationary. The mean of the observations of eight days are available, which are wonderfully accordant. There was no disturbing influence within 60 miles below, and none at all above. The measurements had been checked, up to a few days before, by simultaneous observations at Wilson's Point, 93 miles below, and by the free use of the flanking method at both places. The discharge was also observed at the extreme high stage, 50.3, which was reached the same season; but at that time there were disturbances which rendered the determinations of the discharge less trustworthy, and which palpably affected the gauge height.

The mean discharge for eight days, at a reading of 49.6 on the Arkansas City gauge, was 1 425 206. As computed from the series on page 451, from the data of 1884–85, the discharge at this stage should be 1 431 478.

In comparing the two series just referred to, the first differences represent the increase of the discharge for each 2 ft. of rise. While the two curves are parallel the same differences do not correspond to the same gauge heights, but to the same values of discharge. Thus, in the ascending series, the increase of discharge from 30 to 32 ft. was 64 500. In the descending scale (or rather after the fall) the difference 64 500 does not correspond to the stage of 30–32, but about to that of 32–34. It corresponds, however, in both to a discharge of about 720 000. Hence, in using this method, discharge, not gauge, is to be considered.

The discharges observed at 49.6 may be assumed to be nearly correct. The discharge for any higher stage not too remote may then

be obtained with a very fair approach to accuracy, by applying the analogies deduced from the series on page 451, especially as they have given pretty nearly the correct result above, though extended over nearly 8 ft. of rise. At 50 ft., there should be about 1 445 000; at 50.3, 1 458 500. The mean for six days of the measured discharge actually taken at the latter stage, was about 1 504 000. This includes the extraordinary measurement of 1 667 000. Excluding this, the result was about 1 480 000. Probably this is not far from the truth, but the stage would have been a little higher had it not been for disturbances.

At the stage of 53.3, which is the grade of the present levees, the discharge would be (still starting from 49.6) 1 605 600; at 56, about 1 745 000.

It is now allowable to return to the method of assuming an invariable area and deducing velocities therefrom, which there was doubt about extending to high stages without some check. If the line of assumed velocities is developed beyond the time of the culmination of the rise, and it is followed during the fall, it will be found that this line, like that of the discharges, is approximately parallel to the ascending line (see Fig. 22). It is hardly worth while to give the table from which the several points of the line were derived. If now a straight line is drawn parallel to these lines through the highest point reached in 1885, namely, 41.7, the assumed velocity at any stage will be obtained. At 49.6 it will be about 6.00. The width will be taken at about 3 400. At 41.7 it was 3 380. The increase of height is 7.9 ft.; increase of area, 26 860. Adding this to the assumed datum, 212 000, the assumed area will be at 49.6, 238 860, and the discharge, 1 433 160. At 50.3, the assumed area will be 241 240; velocity, 6.07; discharge, 1 464 327. At 53.3, the area will be 251 440; the velocity, 6.40; the discharge, 1 609 216; at 56, the discharge will be 1 746 154.

Observations of 1893.—The discharge observations of 1893 extended over only a small period, namely, from March 9th to April 8th, and from May 5th to June 7th, covering a range of only 16 ft. on the gauge, from 34.2 to 50.3. For general study they are therefore not so instructive as the series of 1884-85. They have, however, the great advantage of being known to be, in the main, tolerably accurate. They were accompanied by simultaneous observations at Wilson's Point, which serve as a valuable check, especially before crevasses

intervened. They have another advantage, that they were taken at very high stages. It has always been a matter of doubt how far inductions made from medium stages were applicable to extreme high-water conditions. These observations, too, exhibit considerable variations in the cross-section. The datum is taken at 50.3 instead of at 41.7. The datum areas range from 233 252 to 262 008. The standard area will be taken at 242 000, which corresponds pretty nearly to the datum of 212 000 in 1884-85. The data observed during the rise in May are too few, however, and extend over too small a range of the gauge to be a safe guide. The fact that the observations were not continuous prevents tracing the various changes of the bed, although they can be easily imagined. In the considerable discrepancies of the recorded velocities and discharges, it will be safest to take the means of those which were observed while the river was stationary for a considerable period. There were two such times, corresponding to stages of 40.0 and 49.6 respectively. At these dates the following data are found:

Date.	Gauge.	Assumed area.	Assumed mean depth.	Assumed velocity.	Discharge.
March 20th-29th..	40.0	207 479	60.5	4.55	943 931
May 15th-23d.....	49.6	240 022	70.6	5.94	1 425 206

If the line of assumed velocities is supposed to be straight, as before, the velocity at 50.3 will be 6.05, and the discharge 1 464 100. At 53.3, the velocity will be 6.48 and the discharge 1 635 325. The observations actually taken at this stage have already been given. A comparison of the Wilson's Point observations shows no such fluctuations of the discharge as are recorded at Arkansas City. There were crevasses at this time between the two stations, but they had been running for two weeks or more, and there was little change from day to day in their rate of discharge. There should have been then nearly a constant difference between them equal to the combined discharge of the crevasses. The latter were measured twice during the season, but the results are not entitled to much confidence, the conditions being extremely unfavorable and the methods somewhat crude. A crevasse is divided into several sections; at least three direct flows and two reverse flows, caused by eddies, as shown in Fig. 26; and it is extremely difficult to measure their discharge correctly. The discharge at Wil-

son's Point at this time, for the five days succeeding the excessive velocities recorded at Arkansas City, averaged 1 391 658, and for the five days preceding, 1 412 214.

FIG. 26.

Use of Method of Coefficients.—The use to be made of the method of coefficients is to be considered next. In 1884–85, the progression of the theoretical velocities before and after the rise followed nearly straight and parallel lines. So the assumed velocities, on the basis of a constant datum area, followed lines which were nearly straight and parallel, but inclined to the lines of theoretical velocities. The coefficients, when plotted, form curves which have the same general form, but are not exactly parallel (see Fig. 22). In each curve, however, the progression of the coefficients follows very nearly the same law, as might have been expected. The following table gives the values of the coefficients arranged in series, with the differences.

	DURING RISE.		AFTER RISE.	
Gauge.	Coefficient.	First difference.	Coefficient.	First difference.
24	72.4	.....	65.8	.....
26	74.8	2.4	68.6	2.8
28	77.0	2.2	71.1	2.5
30	79.0	2.0	73.5	2.4
32	81.0	2.0	75.8	2.3
34	82.8	1.8	77.8	2.0
36	84.6	1.8	79.8	2.0
38	86.3	1.7	81.8	2.0
40	87.8	1.5	83.7	1.9
42	89.3	1.5	85.5	1.8
44	90.8	1.5	87.1	1.6
46	92.2	1.4	88.7	1.6
48	93.6	1 4	90.2	1.5
50	94 9	1.3	91.7	1.5
52	96.2	1.3	93.2	1.5
54	97.4	1.2	94.5	1.3
56	98.5	1.1	95.8	1.3
58	99.6	1.1	97.1	1.3
60	100.7	1.1	98.4	1.3

At 49.6 ft., in 1893, the coefficient deduced from actual observation, with the assumed data, given on page 458, is 87.6. From either series of coefficients given above, it is seen that a coefficient of this value increases, for an increased height of 3.7 ft. (that is, from 49.6 to 53.3),

by about 2.7. That is, at 53.3 it would become 90.3. The slope would have been about 0.0000651. The mean depth, to datum of 242 000, would have been 74.3. Hence, the assumed velocity would have been 6.28; the area, 252 710; the discharge, 1 587 019.

If, instead of taking an assumed datum area, the observed area, namely 253 774 (mean of 9 days), had been taken, the observed mean depth, 73.3; the observed slope, 0.0000655, and the observed velocity, 5.625, a coefficient of 81.2 would have been obtained, which, at 53.3, should have increased to about 84.5. This should have given a velocity of 6.00, an area of 266 558, and a discharge of 1 599 348. The observed slope was a little greater than the normal for the stage, on account of crevasses below Greenville, which lowered the water line at that place more than at Arkansas City.

As Humphreys and Abbot would say, no closer agreement could be desired, and it is possible that the use of local coefficients may be found serviceable for purposes of approximation or even of accuracy. It would be very rash, however, to come to such a conclusion without further and much more extensive trial. It will appear presently that there must be a separate series of coefficients for each place, which will be subject to variation by any change in the location of the cross-section or any considerable modification of its type. Nowhere are hasty generalizations more unwise, and nowhere are they more frequently brought to grief, than on the Mississippi.

Increase of Coefficient Due to Shape of Section.—It has been remarked that one cause of the increase of the coefficient with increasing gauge height is the deep and narrow conformation of the cross-section. So influential is this cause that all others are as nothing in comparison with it. If, instead of taking the discharge section at Arkansas City, the wide section or bar above had been chosen, a series of coefficients would have been had that moved in an inverse sense, and decreased as the river rose. As this is an important and interesting phenomenon, a few examples may be given. The dimensions of the two sections are taken from the surveys of 1882, and the plottings made therefrom are shown in Fig. 21. As there are no means of ascertaining the changes that occurred from day to day, it will be assumed that the bed of the river underwent no change during the alternation of high water and low water. Whatever change took place would affect the figures only, not the reasoning. The slopes were taken from the mean

results of the observations of 1884-85; the discharges from the mean discharge curve of several years. For the wide and narrow sections respectively the following results are found:

Stage. Feet.	Areas. Square feet.	Mean depths. Feet.	Slope.	Theoretical velocities. Feet.	Discharge Cubic feet.	Actual velocities.	Coefficients.
7.4. Low water of 1884....	51 600 and 86 500	16.8* and 27.0	0.0000375	2.51 and 3.18	221 000	4.29 and 2.56	171 and 80.5
20.....	111 600 and 128 500	21.3 and 39.2	0.000049	3.23 and 4.38	440 000	3.94 and 3.43	122 and 78.4
30.....	139 700 and 161 400	19.1† and 48.0	0.000054	3.21 and 5.09	650 000	4.66 and 4.03	145 and 78.4
35	178 400 and 178 400	23.2 and 52.7	0.000057	3.63 and 5.48	820 000	4.66 and 4.66	127 and 85
42.4‡.....	233 000 and 203 700	30 and 59	0.000059	4.21 and 5.90	1 110 000	4.76 and 5.46	113 and 92.6
50.....	292 000 and 230 000	37.6 and 66.6	0.0000625	4.85 and 6.45	1 450 000	4.97 and 6.30	102.5 and 97.6
55.....	330 750 and 247 250	42.6 and 71.6	0.000064	5.20 and 6.74	1 687 000	5.11 and 6.82	98.2 and 101.2

* In taking the mean depths at low water in the wide section, only the principal part of the section was considered, the isolated pool being left out of account.

† The mean depth is here actually less than a stage 10 ft. lower, because in the meantime the river has spread over a bar 1 500 ft. wide.

‡ The surveys were made at this stage, which corresponds nearly to "bank-full."

Section of Uniform Coefficient.—From these examples it is evident that the commonly accepted idea, on which so many formulas have been based, that the coefficient always increases with the height of the river, or with mean depth, is not a law at all, but in part, at least, a peculiarity attached to narrow sections. A section may be imagined, a sort of mean between these two types, in which the coefficient should remain uniform at all stages from the highest to the lowest. It is indeed possible to construct such a section; it has been done, approximately, in Fig. 27. It is evident that such a section may be conceived as existing at any point in the reach under consideration, and that if it so existed, the discharges would pass at the velocities corresponding to those given by the formula $v = 100\sqrt{rs}$, and at the stages indicated

on the margin. This imaginary section might be used in estimating discharges for higher stages than have been observed. It is a reasonable supposition that at 53.3 ft., for example, the same relation would exist that has prevailed from the stage of 30 to that of 50; that the mean depth of the imaginary section would be 54.5, the slope about 0.0000635, that the velocity would hence be 5.86, the area 272 600 and the discharge about 1 603 000. It is very probable, indeed, that these figures would nearly represent the actual area, velocity and discharge at the Arkansas City section. The neutral section, or section of con-

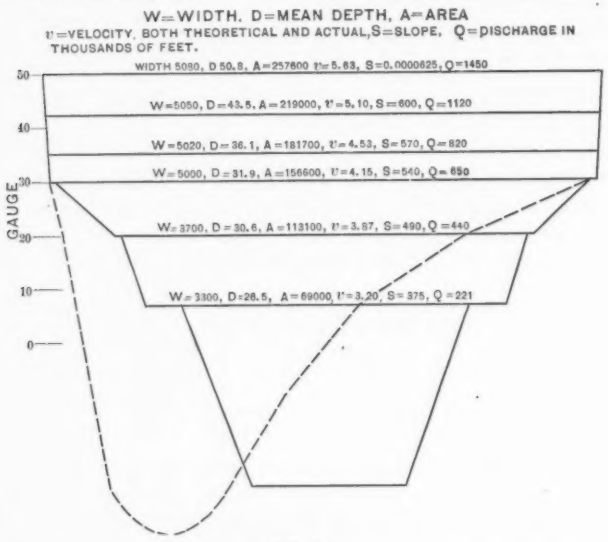


FIG. 27.

stant coefficient, is very nearly a mean of the wide and narrow sections, and is therefore such as would be reached at the top of a great rise, when it had been prolonged sufficiently to bring about an equalization of areas. Indeed, the neutral section is not far from being about the mean section of the river.

For the present purpose, it is hardly worth while to construct such a section with the last degree of accuracy. The shape of the section will depend on the values given the discharges at different stages, and if such values are regarded as satisfactorily determined, higher values may be obtained by arranging the known values in a series and deter-

mining the law of their progression, as has already been done, without resort to an imaginary section.

Discharges at Wilson's Point.—Having examined the data available for the purpose at Arkansas City, Wilson's Point may be taken up. Here there is the disadvantage of having no continuous series of observations extending over low as well as high stages. If an attempt is made to supply the missing data at the lower end of the discharge curve by transferring the Arkansas City discharges of 1884–85, taken at stationary stages, to the gauge readings at Wilson's Point (or rather Lake Providence) at those times, there will be no longer the check which would have been afforded by a series of independent observations, and the results will be substantially identical with those just

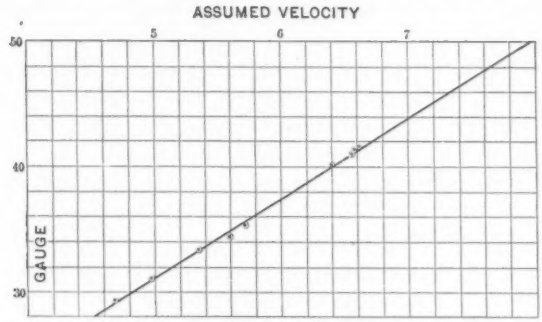


FIG. 28.

obtained. In 1882, there were discharges taken at Hays's Landing, or rather Bass's Landing, 10.4 miles below Lake Providence, which are easily transferred to that point, and are thus comparable with those taken at Wilson's Point, which are referred to the same gauge. The Hays's Landing observations, however, are of little avail, because all the discharges taken at high water were vitiated by the disturbances arising from crevasses and return flows in a wonderful manner. Only about half of the discharge of the river was retained in the channel at that point. The only records which hold out any promise of usefulness are those of 1893, and they have a very limited range.

A datum area of 210 000 may be assumed at 41.8 on the Lake Providence gauge, and the velocities thus obtained plotted to the gauges. The results are not at all accordant, and exhibit discrepancies where

there is no apparent reason for them. For instance, when the river is stationary for several days, and there is no material change of slope, the recorded discharge fluctuates between 972 000 and 930 000, and the reduced velocity between 5.48 and 5.23. Under these circumstances it was thought best to divide the observations into groups and take the means. The result of this proceeding is shown in Fig. 28. The velocities are best represented by a straight line, which, being prolonged, cuts the line of 42.8 (which is supposed to be about equivalent to 50.3 at Arkansas City) at 6.82. This velocity, with the area of 216 580, gives a discharge of about 1 459 000. At 45.7, which may be supposed to be about equivalent to 53.3 at Arkansas City, the velocity will be 7.3; the area, 225 100; and the discharge, about 1 643 000. The observations, however, are too few and their range too small to reason from them with confidence for any considerable interval. It is not certain that 45.7 at Lake Providence would be the equivalent of 53.3 at Arkansas City, for the gauge relations of the two differ widely at different stages, and are not at all consistent.

On the whole, the discharge which passed Arkansas City and Wilson's Point in 1884-85 at a stage of 41.7 on the gauge at the former place and 34.7 at the latter, was about 1 086 000 ft. per second; in 1893, at 50.3 and 42.8* respectively, about 1 460 000. At a stage of 53.3 and 45.7, or whatever is its equivalent at Lake Providence, there would have passed something more than 1 600 000.

These estimates are conservative, and it is probable that the changed conditions which prevail at stages over bank-full would operate so as to increase the discharge by a noticeable amount, perhaps as much as 50 000 cu. ft., as will be shown hereafter.

Maximum Discharge.

It now remains to endeavor to determine the maximum discharge of the river, and the heights which it would have attained at the principal stations had it been confined. For this purpose there are available the observations of 1882, taken at Paducah, Columbus, Helena, Hays's Landing and Red River Landing, and the high-water discharges of the years 1890, 1891, 1892 and 1893. Measurements were made in 1882, 1883 and 1884 of the discharge over banks and through crevasses, and crevasse measurements have been made during each of

* The highest point actually reached was 41.8. Allowance is here made for the effect of crevasses, though the determination is not very certain.

the years of great flood. Of the observations of 1882, the only ones which are available for the purpose of determining the maximum discharge are those of Columbus and Helena.

At these stations, the velocities at extreme high water, in 1882, were usually determined by rod floats.

At Columbus.—At Columbus the length of the rod at high water was generally 37 ft. The mean depth at this stage was about 70 ft., and the maximum about 100 ft. The Francis formula was applied in correcting the measurements, and this would give a coefficient of reduction of about 0.93, for the observed velocities. From the shape of the Columbus

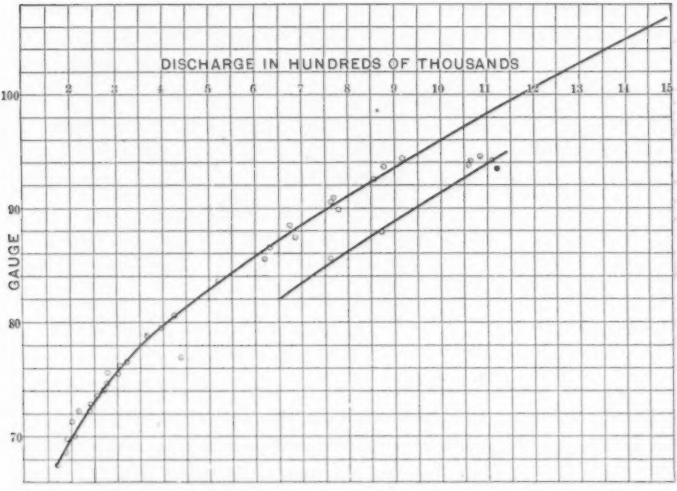


FIG. 29.

curves of vertical velocity at high water, the proper ratio would be about 0.91. Now the highest observed discharge was 1 603 215, on a rising river, at a stage 1.5 less than the maximum. A few days later, after a fall had set in, a discharge of 1 558 323 was recorded, at a stage only 0.17 less than the maximum. The slopes in these cases were 0.0000895, 0.0000960 and 0.0000913, so there seems to be no reason why the first discharge should have been in excess of the last.

If the discharge curve is plotted, it will be found that the data for the ascending branch are very few (see Fig. 29). So far as they go,

they correspond very fairly to those of the descending branch, which are pretty numerous, and afford the means of determining the curve with reasonable accuracy. Here, as at Helena, later, the means of the observations near the same stage, on a falling river, have been taken. It is a little flatter than the Arkansas City curve. The low-water part of the curve seems to follow a different law of progression from that of the high-water part, a frequent occurrence; and above the overflow line, which is fixed by the observers at 95 ft. on the gauge, there is still a different law, from causes which are now clear.

The series of values found for the branch of the curve pursued after the rise are as follows:

Gauge.	Discharge.	First Difference.	Second Difference.
78	355 000	.....	.....
80	415 000	60 000	.....
82	478 000	63 000	3 000
84	544 000	66 000	.....
86	613 000	69 000	.....
88	685 000	72 000	.....
90	760 000	75 000	.....
92	838 000	78 000	.....
94	919 000	81 000	.....
96	1 103 000	84 000	.....
98	1 090 000	87 000	.....
100	1 180 000	90 000	.....
102	1 273 000	95 000	.....
104	1 369 000	96 000	.....
106	1 168 000	99 000	.....

Above 95, a well determined series of measurements gives the increase of discharge from the stage of 100 to that of 102 as about 130 000 cu. ft. Taking the mean of nine observations at the nearly stationary stage reached from January 25th to February 6th, it will be found that a gauge reading of 99.96 corresponded to a discharge of about 1 394 000. At this rate, the discharge at the highest point reached, 102.8, should be about 1 573 000. This value depends, of course, on the correctness of the determinations. If the errors in them are merely in the nature of fluctuations, probably they will be corrected by using means, as has been done. If the quantities are all too large or too small, owing to common sources of error, the result will be affected in the same way. Relatively to one another, the observations seem to be fairly consistent.

It was estimated that a considerable volume of water escaped over the banks, and did not pass Columbus in the channel at all. The

measurements of this quantity made in 1882 and in 1883 are so discrepant, and the matter is so surrounded by complications, that it can be discussed more conveniently in connection with the subject of escapes and returns from reservoirs or across points.

It is now in order to see how these conclusions tally with others, made at different times and with more accurate means. In the floods of 1892 and 1893, high-water discharges were taken at Columbus, and in the latter year at New Madrid also. In 1892, the Cairo gauge reached 48.3, and in 1893, 49.3, respectively 3.6 and 2.6 ft. less than in 1882. In 1892, the mean for the five highest days was 1 379 770. The gauge reading for the same time was 43.0 on the Belmont gauge, equal to 101.0 on the old Columbus gauge. In 1893, the mean of nine days was 1 493 507, for a gauge height of 43.7 at Belmont, and of 49.2 at Cairo. At New Madrid, the discharge for a mean of seven days, was 1 474 048. The observations at Columbus were taken with double floats at mid-depth, and are, no doubt, too large. Even at New Madrid the same remark may hold good. The estimate of the discharge given above is the mean of the meter and the float observations. The meter was placed at 0.6 depth and the floats (double) at mid-depth, and no correction was applied for either. The velocities found by floats, however, were almost uniformly the smaller of the two.

For a height equal to that attained in 1882, namely, 44.8 on the Belmont gauge, the discharge would no doubt have been about 1 540 000.

At Helena.—At Helena, in 1882, the highest recorded discharge is 1 558 485, taken with the meter. There is an observation showing a discharge a little greater than this, taken a few days later, but it was made with rod floats, and the area was interpolated. The correction, by Francis's formula, if the rods were of the same length as those used at Columbus, would be 0.95, the proper correction, according to the curves of vertical velocity, being 0.93.

It is doubtful, then, from the record, whether so great a discharge as 1 550 000 passed. In 1893, at an average stage of 47.8, a mean discharge of 1 527 501 is recorded.* The stage reached in 1882 was only 47.2. There were some circumstances, however, that indicate the possibility of an increased discharge at Helena, even at a lower stage.

* So taken with meters at 0.6 depth. Discharges taken with double floats on the same days gave a little more.

Within 20 miles below Helena, there were several considerable crevasses, discharging some 150 000 cu. ft. per second, which must have lowered the water line a good deal, while giving an increased slope. It is perfectly possible, then, that the recorded discharge was genuine. Even in 1893, the levees of the White River basin extended only about 22 miles below Helena, and there was a gap of some 25 miles next succeeding. It is not to be doubted that an important influence was exerted on the Helena gauge by the depression of the water line below that station thus produced.

There is also a record of a discharge of about 1 310 000, at a stage of 45.7. That a discharge of 200 000 ft. more should be passed at an additional height of only 2 ft. indicates a great disturbance of the ordinary relations.

Influence of Saint Francis Basin.—The influence of the vast unleveed front of the Saint Francis Basin on the discharge and on the gauge height at Helena and at points below is indeed a question which is not well understood, and upon which opinions have been divided. It has been reasoned out on various lines. It was for a long time maintained by levee engineers, and by the author among others, that the effect of leaving the front of this great reservoir open was rather to increase the discharge and the height of the flood line at the foot of the basin. The argument in support of this view ran somewhat thus: Given a river inclosed by levees, on the one side of which is a great basin like that of the Saint Francis. Given a flood, the greatest that can be restrained by those levees. Then, near the height of the flood, the river-bed, thus enlarged artificially, is doing its whole duty, and carrying a discharge of 1 500 000 ft. Now let a breach be made in the levee bordering the reservoir. A volume, say of 300 000 cu. ft. per second is abstracted from the mass moving in the channel, and the discharge is reduced to 1 200 000. Then the stream is doing less than its duty. Now let the quantity of water lately abstracted be returned at the foot of the basin suddenly, as it is always returned, at a rate faster than that at which it was withdrawn. Then, as the river was lately doing less than its duty, so now it will have to do more, and the discharge at the foot of the basin will be greater than it would have been had the levees remained unbroken. So likewise the flood height will be increased by reason of the additional discharge. Not only so, but of the mass of water thus returned, a part has a low velocity and is projected into the

river at an unfavorable angle, so that a rise of the water surface will be produced until sufficient height is acquired to confer the requisite velocity.

This position was disputed some years ago by Captain Rossell, and has lately been attacked with much ability by Captain Townsend.* These engineers draw their arguments from the relations known to prevail between the different gauges. For instance, a comparison is made of the Cairo and Helena gauges, care being taken to select the crests of rises or the extreme low points of falls, so as to readily identify the corresponding points, and not be forced to apply a somewhat doubtful time interval. It is found that at all stages from zero to bank-full, there is, on the whole, a striking parallelism between the gauges, Helena being usually a little the higher. The relation is somewhat variable, depending mainly on the sharpness or duration of the rise, but if the latter be long enough to be propagated uniformly down the river, Helena will be about 0.5 ft. ahead of Cairo. At stages beyond bank-full, however, this relation ceases. Cairo then becomes the higher, and this by a constantly increasing quantity with the magnitude of the rise, so that in 1882 it was 4.7 ft. above Helena. In 1893 it was only 1.4 ft. above. This remarkable departure from previously existing relations is ascribed by Captain Townsend to the influence of the Saint Francis Basin acting as a reservoir. According to this view, the water abstracted from the upper end of the basin is retained in the reservoir a sufficient time to allow the crest of the flood to pass Helena before it is returned. When it is returned, it is not in sufficient quantity to raise the flood line to the height which it would have attained in a confined state.

This argument is very forcible, and it has the merit of being based on accurate and indisputable data. Discharge observations and measurements of flow over banks, through crevasses and across points, are attended with many elements of error and uncertainty. Gauge records, however, may be kept with any desirable degree of exactitude. There are no gaps in them. Errors, if there are any, are always subject to correction, and the relations between neighboring or even distant gauges maintain a considerable degree of consistency when allowance is made for disturbing causes, such as duration of rise, effect of tributaries, crevasses, etc.

* "Report of the Mississippi River Commission for 1894," p. 2967, *et seq.*

The plain consequence to which this reasoning leads is that if the front of the Saint Francis Basin were completely sealed by levees, the gauge at Helena, in long-continued floods, would eventually attain a height at least equal to that reached at Cairo.

Captain Townsend does not attribute the present diminished gauge height at Helena wholly to the Saint Francis Basin, but ascribes a part of it to the condition of the White River Basin. The facts

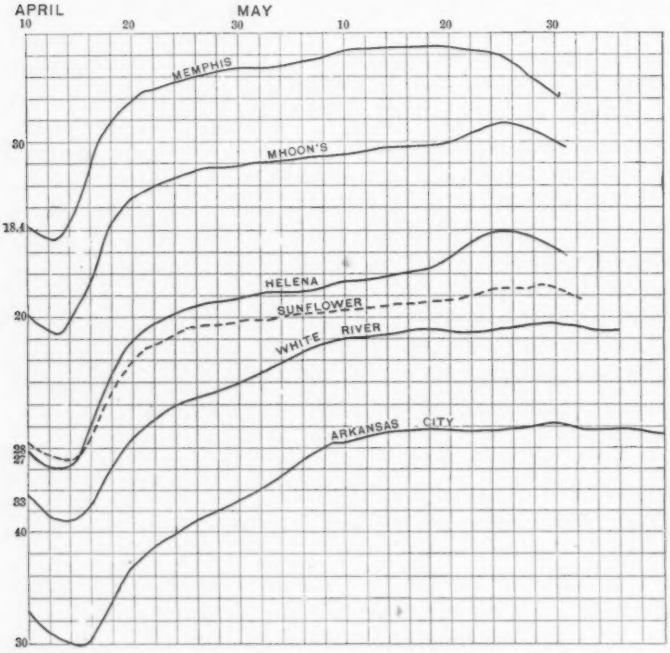


FIG. 30.

developed in the study of the Helena discharges point to the same conclusion. If the discharges can be relied on at all, the quantity of water passed at Helena in 1893 was about equal to that which passed Columbus and New Madrid, increased by the discharge of the Saint Francis River, which was at a low or medium stage. It was merely the height, then, and not the discharge, which was affected in this year.

Return of Flood Water.—The manner of the return of the flood water

from the Saint Francis Basin may best be studied by referring to the hydrographs in Fig. 30, for the high water of 1893 of Memphis, 76.5 miles above Helena, of Mhoon's Landing, 29.2 miles above Helena, of Sunflower Landing, 46.2 miles below, and of the mouth of White River, 91.7 miles below. It will be seen that the Helena gauge moved along, nearly parallel with that of Memphis, until the river reached its height at the latter place and had even begun to fall, when, about May 19th, at Helena, a rapid upward movement began at the rate of 0.5 ft. per day. This rate prevailed for about a week. The rise from May 17th to May 25th was 3.3 ft. The gauge at Mhoon's Landing sympathized with this movement, either because the return flow began so high up, or from the effect of the back water. From the appearance of the hydrograph it was probably the latter. The discharge increased, during the same time, from about 1 300 000 to more than 1 500 000. This is a greater rate of increase than is usual, even at such high stages, and points to extraordinary causes. The most prominent of these is the great increase of slope produced, first, by the sharp elevation of the water surface at Helena, and second, by the lowering of the water surface below Helena by the gap in the Arkansas levees acting as an outlet or waste weir. The amount of the increase of slope is very fairly deduced from the changes of general slope, as revealed by the comparison of the adjacent gauges below. Sunflower Landing, the next gauge station below, is directly opposite a part of the gap, and the relations between the Helena and the Sunflower gauges represent more closely than usual the local conditions. Assuming a fixed datum area of 220 000, the following table may be constructed:

Date.	Gauge.	Assumed area.	Assumed mean depth.	Slope, Helena-Sunflower	Assumed velocity.	Assumed theoretical velocity.
May 17...	44.6	203 526	40.8	0.0000738	6.45	5.49
25...	47.9	220 000	44.0	0.0000816	6.95	5 99

The coefficients were 117.5 and 116.5. The local slopes were measured, and are given as 0.0000648 and 0.0000836 respectively.

Height to be Attained at Helena.—Can the height which the gauge would have attained if there had been no disturbing influences above or below be deduced from the Helena discharges? If so, it can only be by extending to high stages the analogies developed at heights up to bank-full.

For this purpose the observations of 1890, 1891, 1892 and 1893 are of no use, for none of them were taken at stages below bank-full. The records of the years 1882 and 1884–85, however, cover all stages from low water up.

The observations of 1882 exhibit many discrepancies, as might have been expected. They were mostly made after the flood, so that it is impossible to take an ascending series of discharges, as in 1884–85 at Arkansas City. It is assumed that the descending series moved nearly parallel to the ascending, as usual; the law of the progression of the discharge will be found even from these imperfect data. The absolute quantity of the discharge, however, will not be obtained. It may be considerably more than that indicated by the descending series, as it would be less than that shown by the ascending series.

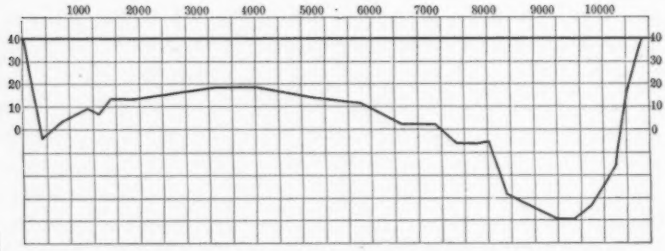


FIG. 31.

It was remarked in relation to the discharge at Columbus that the lower part of the curve followed a somewhat different law of progression from that which governed the upper part. This is found also to be the case at Helena and at Arkansas City. It is nothing more than might have been expected, when the shape of the cross-section of the river is considered, and especially of the wide sections which appear to control the height of the water. Fig. 31 represents the cross-section of the wide reach above Helena. Here, as at Arkansas City, at a stage about 20 ft. below bank-full, or at a reading of about 20 ft. on the gauge, the river spreads over an immense sand bar 1 500 ft. wide at the latter place and 4 000 ft. wide at Helena. Previously to this time it has flowed in a comparatively narrow and deep channel of nearly the same type as the discharge section. There is no reason why it should follow the same law after this change. On the contrary, the rate of

the discharge should increase, in consequence of the great width and shallow depth of the new part of the section, the relatively rapid augmentation of the velocity and the doubly rapid increase of area. The rate of increase, however, will be slow at first, on account of the small velocity of the new section for the first few feet after it has been covered. If these things be so, then the discharge curve in such localities will really be a compound curve.

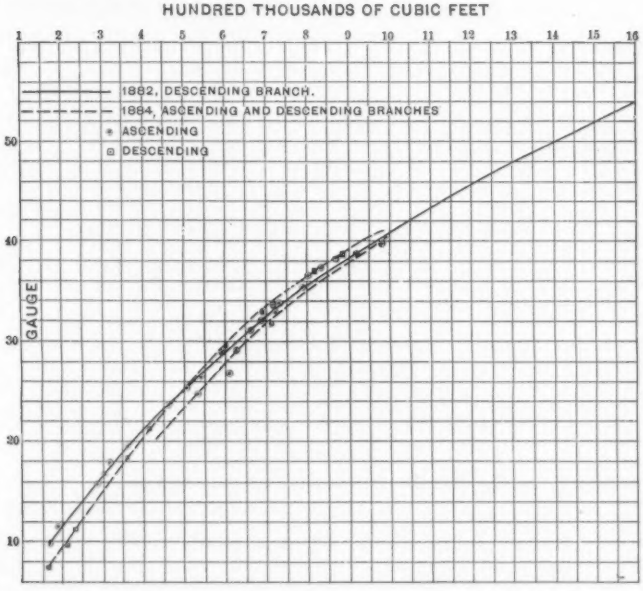


FIG. 32.

Helena Discharge of 1882.—In the construction of the following table, the observations made at nearly similar stages have been combined and the means taken. For instance, all the observations taken between 38 and 39 ft. on a falling river, eight in number, have been used for the figure of 916 500, given in the plot as corresponding to the mean stage of 38.6. As it is the object to trace the descending branch of the curve, none of the measurements made during the progress of a rise have been used The table shows the progression of the discharge in the descending branch from low water to bank-full, and the extension of the same

relations to stages higher than have ever been reached in a confined river. The principal break in the curve seems to occur at about 30 ft. The observations of 1884–85 also have been plotted and are given in Fig. 32. It will be seen that at low stages they depart somewhat from the curve of 1882, while at high stages they exhibit nearly the same law of progression. It seems hardly worth while to tabulate them.

Gauge.	Discharge.	First Difference.	Second Difference.
10	171 000	.....	.....
12	208 000	37 000	.....
14	247 000	39 000	2 000
16	288 000	41 000	.....
18	331 000	43 000	.....
20	376 000	45 000	.....
22	423 000	47 000	.....
24	472 000	49 000	.....
26	523 000	51 000	.....
28	576 000	53 000	.....
30	631 000	55 000	.....
32	690 000	59 000	4 000
34	753 000	63 000	.....
36	820 000	67 000	.....
38	891 000	71 000	.....
40	966 000	75 000	.....
42	1 045 000	79 000	.....
44	1 128 000	83 000	.....
46	1 215 000	87 000	.....
48	1 306 000	91 000	.....
50	1 401 000	95 000	.....
52	1 500 000	99 000	.....
54	1 603 000	103 000	.....

In 1882, at stages above 40, there is exhibited a manifest tendency to depart still further from the law of progression which the river seems to have observed while within its banks. At 41.6 on the gauge, a mean of 12 consecutive observations gives a discharge of 1 101 000 cu. ft., which is considerably in excess of what it should have been with the relations prevailing at that time. Probably this is attributable to an escape of water below Helena. These measurements were taken after the flood, and there were several deep crevasses still running at this stage which no doubt operated to lower the water surface at the latter point. This, indeed, is clearly indicated by the gauge at Malone's Landing, 49.2 miles below, which was nearly 2 ft. lower than its ordinary relation to the Helena gauge would prescribe.

It is not safe to reason from measurements taken at a stage of 30-40 ft. to stages of 50 ft. or more, without some facts to check up on. From the nature of the case, there are no high-water discharges at Helena which are not affected by the very causes which it is sought to eliminate. Reasoning from such data as are available, if it is assumed

that at about 40.5, in 1885, a discharge of 1 015 000 passed as recorded, then by the rule of progression which seems to prevail, there should be a discharge of about 1 430 000 at a stage of 50. By the same reasoning a discharge of 1 500 000 should pass at a gauge reading of about 51.4. It is recorded that such a discharge passed in 1893 at 47.9, indicating, so far as this argument may be relied on, that the flood line had been depressed by high-water influences 3.5 ft. So far, then, Captain Townsend's conclusions are confirmed, and the confinement of the river by the closure of the Saint Francis and White River Basins will have the effect of augmenting the gauge height at Helena.

Will this increased flood-height be propagated down the river, and, if so, how far and to what extent? The answer to this question will depend, in part at least, on the importance of the White River Basin as an agent in affecting the water plane.

Influence of White River Basin.—Even at the date of this writing, there is a gap in the line of levees of the White River Basin some 10 miles in length, which, however, will be closed, no doubt, in two or three years. This gap is sufficient to discharge 200 000 or 300 000 cu. ft. per second at high water, and to depress the flood plane in its vicinity 3 ft. or so.* Such a steepening of the slope at Helena must have a very considerable influence on the velocity at the latter point, and consequently on the flood height. It is not impossible that a great part of the effect usually attributed to the Saint Francis Basin may be more properly due to the outlets below Helena. It is well known that the outlets below Arkansas City in 1886 and 1887 had a notable effect in lowering the high-water line at that place. In 1887 a discharge measured as a mean of 1 440 000 cu. ft. passed Arkansas City at a height of 46.6 ft. Such a discharge corresponds normally to a height of nearly 50 ft.

The water which is poured over the bank through the gaps into the White River Basin soon returns to the Mississippi, usually through the channel of White River itself, if that stream be not very high. If it be in a state of extreme flood, a part of its discharge is transferred to the foot of the basin by overflow, and enters the Mississippi along the line of the Cypress Creek levee. As the basin is only one-sixth the size of the Saint Francis Basin, its effect is small com-

* Captain Roessler estimates the "potential high-water line," or water line as it would be if the stream were entirely confined, as 2.7 ft., above the actual flood line in some localities.

pared to that immense territory, though still of great importance in several respects.

It was seen that the discharge at Columbus in 1882 was probably 1 600 000 cu. ft. or more, including the volume which passed in the swamp. Now the discharge which passed Helena was only 1 500 000 ft. or so. Does not this indicate that a part of the water was held back in the Saint Francis Basin? It does not. The levees of the upper part of the Yazoo Basin, from the hills below Memphis to Glendale, opposite Helena, were in a ruinous condition, and the volume of water which poured over them was estimated in 1882, and again under circumstances almost precisely similar in 1883, at 290 000 cu. ft. per second. As has been stated, these measurements are always excessive; but allowing only half of the above estimate, it will account, not only for the 100 000 cu. ft. of difference between Columbus and Helena, but also for the discharge of the Saint Francis River proper, estimated at about 50 000 cu. ft.

What Height Could be Reached at Cairo?—If it be true, as seems probable, that in a confined river, in prolonged floods, the river would rise to a height on the Helena gauge equal to or greater than that attained at Cairo, what would now be the altitude which that standard of the river would reach? For even at Cairo the river is not closely restrained. There is a considerable flow over the bank at Bird's Point, in Missouri, just opposite, which was measured in 1883 and reported at 216 000 cu. ft. per second. It may reasonably be supposed that such a loss of volume as this would have a material effect on the water line, and indeed it has been maintained that it would depress it 2 ft. or more.

Water Escape Below Cairo.—Now, in 1883 the river attained a higher point at Cairo than in 1882 or in any other year, before or since. The records for the three remarkable seasons of 1882, 1883 and 1884, being the means for six successive days, are 51.55 ft., 52.03 ft. and 51.72 ft. The measurement of the flow across Bird's Point in 1883 was taken at a stage of 52.08 ft., so it fairly represents the maximum. Of this volume 111 000 cu. ft. per second were returned within 8 miles below Cairo, and 37 000 cu. ft. more at about 13 or 14 miles below; but there was reported a great outflow in the bend below the latter point, estimated at more than 300 000 cu. ft., most of which was returned again about Columbus; but the records give a large net escape until New

Madrid was reached, when it all or nearly all came back into the main river.*

Discharge Across "Points."—The discharge of water across "points," or peninsulas unprotected by levees, and the return of the same water in the bends below, sometimes reaches enormous proportions. These great movements have not often been carefully estimated. They were roughly measured in 1883, in the hydrographic reconnaissance of the Saint Francis front for the purpose of estimating the water escape of that year, and they seem sometimes to have attained the astonishing figures of 300 000 or 400 000 cu. ft. per second. In 1893, the discharges across Ashbrook's Point and Carter's Point were measured by Captain Townsend and found to be (at the highest stage) about 200 000 and 300 000 cu. ft. respectively.

Disturbances of Level.—It would seem that the transference of these enormous masses of water could not take place without serious disturbances of level. The abstraction of 200 000 cu. ft. of water per second permanently would reduce the water level at the point where it occurred at least 2 or 3 ft., and the injection into a stream of as much water from a tributary should raise it by a like amount. It is easily seen, however, that where the outlet and the return inlet are only 10 or 12 miles apart, such processes are incompatible with one another. The whole fall of the river in 14 miles, in the neighborhood of Carter's Point, is only about 4 ft., and the river would thus be reduced to an absolute pond. No such phenomenon occurs. The slope is disturbed indeed, but not to any such extent, which would involve an impossibility. In 1893 the difference of water level between the upper and the lower sides of Carter's Point at high water, when the movement of 300 000 cu. ft. per second took place across the peninsula, was 4.4 ft. in a distance of 15.5 miles, or 0.284 ft. to the mile, a good deal less than the normal for the whole reach, which is 0.34, and corresponding to a disturbance of level of 0.85 for the 15.5 miles; provided, indeed it could be proved that the normal relation would have existed in this particular portion of the reach. This cannot be assumed without direct evidence. In a smaller flood, 1891, however, the relation was 5.3 ft. for the same distance, or a slope of 0.342 to the mile, namely, about the normal. There was a considerable discharge across the point even in 1891. It may be concluded that the difference of level intro-

* "Report of the Mississippi River Commission for 1884," pp. 2623 *et seq.*

duced by the combined influence of the outflow and inflow was about 1 ft. By the time Greenville was reached, 4 miles below, the slope had been restored and even a little more, the fall in the 4 miles being 2.4 ft.

Back Water.—The phenomenon of the rising of the water surface on the lower side of overflowed "points" in great as compared with small floods has been observed by levee engineers, and allowance is or should be made for it in establishing grades. The means by which the slope is so adjusted as to carry the discharge is what is commonly known as "back water"; that is, the effect of the ingurgitation into the stream on the lower side is to raise the water surface, and this immediately acts to check the velocity above, and hence to increase the cross-section, and this to a considerable distance. Without discussing the general question of back water, which has been made the subject of a good deal of investigation, it will suffice to look at the examples presented by recent floods in the Mississippi. One example has been given already on page 471, in the effect of the return flow at Helena upon the gauge at Mhoon's Landing, 29.7 miles above. Still more striking illustrations were afforded in 1882 and 1884 by the great return flow at Vicksburg from the Yazoo Basin, then only partially leveed. The effect of this tremendous engorgement was very perceptible at Hays's Landing, 45.6 miles above Vicksburg, as will be evident by an inspection of the hydrographs, given in Fig. 33. From the Greenville hydrograph, it is plain that the river would have been falling at Hays's Landing on March 18th, 1884, had it not been for the return wave at Vicksburg, the influence of which not only kept it from falling, but raised it about 1 ft. The curve of 1882 was somewhat affected by local conditions. Especially, the sudden fall at Hays's Landing on March 21st and at Vicksburg on March 22d was due to several great crevasses which occurred on the former day, some

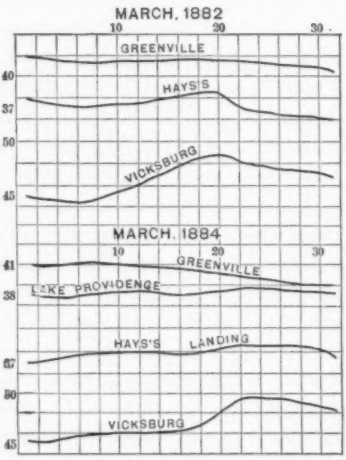


FIG. 33.

10 miles below Hays's. As the gauges at Greenville and at Hays's generally move nearly parallel, it is to be inferred that, instead of rising 1 ft. in 1884, the river would have fallen about 1.5 ft. up to March 25th, had it not been for the back water. The effect of the latter has been, then, to raise the water surface at Hays's about 2.5 ft.

The relation between the Vicksburg gauge and that at Greenville is not so well established. It is affected by the stage of the Yazoo River and by other causes, but it is safe to infer that the gauge reading at Vicksburg, from the discharge which passed Greenville in the channel, would not have exceeded 44 ft. There was a rise, therefore, from the return flow of the Yazoo Basin of at least 5 ft., and half of this effect was produced 45 miles up the river. An effect of 2 ft. was produced at Lake Providence, 57 miles above Vicksburg, and it is thought to have been distinctly perceptible as high as Longwood, in Mississippi, 98 miles above Vicksburg. It should be observed, however, that the greater part of the return pour was discharged, not exactly at Vicksburg, but near the mouth of the Yazoo, 10 miles above.

Effect of Outlets and Inlets Below Cairo.—The effect produced at Cairo by the flow across Bird's Point for a few days would not have been much had the water restored to the channel remained there. As it was, the great outlets in Lucas Bend, 15 miles below, the water from which was partially returned near Columbus, 21 miles below Cairo, and the most of the remainder 40 to 45 miles below Cairo, must have in some measure offset the return below Bird's Point.

As to the quantity discharged over the latter point, there is room for much doubt. The same observer, in 1882, at a stage 4.5 ft. less, found the bank of Bird's Point exposed, and no escape at all from Commerce, in Missouri, to a point 14 miles below Cairo.* It seems hardly credible, therefore, that the Point should have been overflowed for a distance of 2 or 3 miles, to a depth varying from 9.5 to 11 ft., even at the highest stage of 1883. From the very crude methods employed, the results can hardly be accepted even as approximations.

The slope Cairo-New Madrid in 1882 was 0.369 to the mile; in 1883, 0.370; in 1880 (gauge at Cairo, 44.60 ft.), 0.358; in 1893, 0.358. Therefore the gauge height at Cairo in 1882 was not diminished below its usual relation, but rather increased. In 1882 the New Madrid

* "Report of the Mississippi River Commission for 1884," page 2597.

gauge * stood at 41.49 ft., while that at Cairo was 51.87 ft., a difference of 10.38 ft. In 1883 the gauges were 52.17 ft. and 41.74 ft., a difference of 10.43 ft. In 1884 the figures were 51.79 ft. —41.50 ft. = 10.29 ft.; in 1880 they were 44.60 ft. — 34.70 ft. = 9.90 ft.; in 1886, 50.98 ft. —40.38 ft. = 10.6 ft.; 1892, 48.3 ft. — 37.7 ft. = 10.6 ft.; in 1893, 49.3 ft. — 38.7 ft. = 10.6 ft. New Madrid is chosen as the standard of comparison because all or almost all of the water which enters the Saint Francis Basin below Commerce returns by the New Madrid ridge and passes in the channel at New Madrid. The relation between the Cairo and the New Madrid gauges is shown in Fig. 34. The difference between them increases with height, but at high stages it seems nearly uniform. In 1892 and 1893 there could have been little or no escape at Cairo; and the fact that the same relations prevailed between the two gauges in the three great floods of 1882, 1883 and 1884 as in 1892 and 1893 leads to the inference that there was no extraordinary disturbance at Cairo, unless indeed there was a similar disturbance at New Madrid. Now, the loss of volume below New Madrid, either in 1882 or 1883, was not considerable. There was no great outflow, even as shown by the observations of 1883, for about 50 miles below that station. There is reported, indeed, a great inflow just above it; but without a corresponding outlet below, there should have been no depression of the water line at New Madrid. A difference of 11 ft. between the Cairo and New Madrid gauges is probably not far from the truth. This relation would give for a confined river, in 1882, a gauge height at Cairo of 52.5 ft.

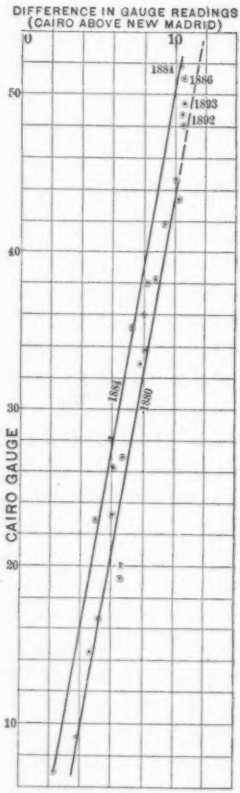


FIG. 34.

* Deduced from the high-water observations of 1882. The gauge here mentioned is, in strictness, the Morrison's Landing gauge, which is 1.3 miles above New Madrid. Lately (1893) a new gauge has been set up at New Madrid proper, and the Morrison's Landing gauge discontinued; 38.7 on the latter corresponds to 38.1 on the former.

Discrepancies in Estimates of Escapes Over Banks.—The estimates of flow over the banks in 1883 seem excessive from another point of view. It was seen that in 1882 the quantity of water reported as passing at Columbus outside of the channel was about 86 000 cu. ft. In 1883 it is reported at 300 000 cu. ft. or more. The difference in the mean gauge readings at Cairo was 0.5 ft. The reading at New Madrid was 0.25 higher. Very nearly the same conditions prevailed in the Yazoo Basin and the Saint Francis Basin. The gauge at Helena was lower by 0.3 ft. Therefore the discharge could not have been considerably greater in 1883. Therefore either the estimate of 1882 was too small or that of 1883 was too great. By taking the discharge observations at New Madrid in 1893, it will be found that a stage of 41.5 ft. on the Morrison's Landing gauge (40.9 ft. on the present New Madrid gauge) corresponds to a discharge of about 1 600 000 cu. ft. It is therefore fair to infer that this was about the actual discharge which passed Cairo in 1882, and that the gauge height at the latter point, if the river had been confined, would not have exceeded 52.5 ft.

Relations between Helena and Arkansas City.—At Helena, according to Captain Townsend's reasoning the height would have been a little greater, about 53.0 ft. From the estimate given on page 475, a very similar conclusion is drawn. Now, what will this correspond to at Arkansas City, with the White and Arkansas Rivers at low and at high stages respectively?

It is very difficult to eliminate from the evidence the effect of the discharge from the tributaries. Taking only well defined and prolonged flood waves, and using the averages for several days, it will be found that for low stages of the White and Arkansas Rivers, the Arkansas City gauge usually stands about 0.4 ft. or 0.5 ft. above Cairo, or in fact nearly equal to the Helena gauge. When the tributaries are in great flood, Arkansas City has been known to be as much as 4.7 ft. the higher. At the same time, it was 3.9 ft. higher than Helena. This was in May, 1885, when the Mississippi was at a stage less than bank-full. Generally, at high stages, but with a confined river, it may be said that, with equal discharges, Arkansas City stands a little higher than Cairo. In 1893, for instance, with a discharge of about 1 450 000 cu. ft. at Cairo, the gauge stood at about 49.25 ft. for seven days. At Arkansas City, for a like discharge, it stood at 50.0 ft.

Effect of Floods from the White and Arkansas Rivers.—Now, what would be the effect upon the gauge at the latter place of a great flood from

the two tributaries—for they nearly always come out together? As a general rule, the upper waters of the Arkansas River contribute little to the Mississippi, flowing through an arid and sandy region. It is from the abundant rainfall upon the southern slope of the Ozark Mountains that the damaging floods derive their origin, or at least receive their principal increment. The White River drains the northern slope of the same range; and the storms which produce a freshet in one bring out the other also, as well as, ordinarily, the Saint Francis. The flood of May, 1885, just mentioned, reached 28.60 ft. at Little Rock (engineers' gauge)* and 25.60 ft. at Jacksonport. This should have corresponded to a discharge, as measured, of about 330 000 cu. ft. for the two rivers. This and the increment from the Saint Francis, whatever it may have been, raised the gauge at Arkansas City about 4.7 ft. The greatest flood which has occurred since the time of accurate observation, and one of the greatest ever known in these rivers, took place in May, 1892. This flood reached 31.2 ft. at Little Rock, and 30.4 ft. at Jacksonport. The measured discharge at the former place is given as 456 000 cu. ft., but this is believed to have been excessive. Judging from the observations of 1879 and of 1893, it should have been, for an average of several days, more than 300 000 cu. ft. In White River it was about 170 000 cu. ft. The result of this addition to the discharge of the main river should have been an increase of height (reasoning from the experience of 1885) of about 6 ft. at Arkansas City over Cairo. The reading at the latter station was, for a mean of six days, 48.0 ft. The height actually recorded at Arkansas City was 50 ft. There was a loss of water estimated at 300 000 cu. ft. per second around the end of the levee above Arkansas City, and there was a considerable crevasse 14 miles below that place, discharging, it was said, about 80 000 cu. ft. In 1893, the discharge from the tributaries, at the height of the flood, may be estimated at 100 000 cu. ft. The loss around the end of the Cypress Creek levee is given as 97 000 cu. ft. Consequently the gauge stood at nearly its normal relation to Cairo.

In 1882, at the crest of the flood, the White River was discharging about 90 000 cu. ft. and the Arkansas a little less.

At High Water, the Tributaries Discharge Partly Through the Basin.—It must not be forgotten that the two great tributaries which empty above Arkansas City do not pour the whole of their discharge at high

* There are no less than three gauges at Little Rock, and they are frequently confused.

water directly into the Mississippi. At such times the bank is overflowed to the depth of several feet, in some places 15 ft. or 20 ft., and the entire White River Basin is converted into a lake of 1 000 square miles area, or, including portions of the valleys of the two rivers within the reach of back water, of 1 200 square miles. Through this lake a part of the discharge of the two rivers pours, the greater portion through the mouth of White River, the rest through the old mouth of the Arkansas. A considerable fraction of the combined river discharge and overflow water, however, goes to swell the volume of the lake, and is carried with a gentle current to the foot of the basin, a few miles above Arkansas City, where it is finally emptied into the main river. The lake partially acts as a regulating reservoir, and sharp discharge waves are modified by it.

Effect on Gauge Height of Discharge of Tributaries.—Why a discharge of the tributaries of 330 000 cu. ft. in May, 1885, should produce an increase of height at Arkansas City over Helena of only 3.9 ft. remains to be examined. It has been seen that at high stages about 50 000 cu. ft. increase of discharge corresponds to 1 ft. increase of height. An addition of 330 000 cu. ft., therefore, should produce a difference between Helena and Arkansas City of 6.6 ft.

It has long been known that the discharge of a stream after the reception of a tributary is less than the combined discharges of the two. This is principally ascribable to two causes. First, the freshets from the tributaries do not reach the main stream at the crest of its own principal rise; and second, the channels, and, still more, the flood plains of the minor streams, where such exist, have a considerable reservoir capacity, by which the excessive discharges of the latter, always comparatively brief, are held back and distributed over several days.

The most formidable freshets of the White and Arkansas Rivers, as has been stated, owe their origin to the rain which falls on the northern and southern slopes of the Ozark Mountains and their outliers. These rains are sometimes very heavy. In May, 1892, at one station in the Arkansas Valley, a fall for the month of 18.5 ins. was reported, and the general depth of rain on both slopes of the mountains was about 12 ins.

The cyclonic storms from which this precipitation proceeds now continue on their way to the upper waters. They drench the valley of the Ohio in spring or of the Missouri and upper Mississippi in summer, one or two days after they have done their work in Arkansas.

Now the floods of the lower tributaries reach their mouth in three or four days. Those of the upper rivers do not get to the same point for nine or ten days ; or if they have to pass through the Saint Francis Basin, not for three weeks. Consequently the freshets from the White and Arkansas have done their worst before the main rise comes, and when it reaches their mouth, it finds them falling.

In 1885, the Arkansas attained its maximum at Little Rock on April 27th to 29th; the White River at Jacksonport on April 28th. The Mississippi reached its height at Helena on May 4th, and the crest of the rise reached the joint mouth of the two tributaries about May 6th. By that time, the minor rivers had fallen several feet, and their joint discharge probably did not exceed the 200 000 cu. ft. accounted for by the rise.

Unfortunately, no discharges were taken at Arkansas City during the second flood of 1885, in May, but reasoning from the measurements made during the previous rise in January, the masses of water which passed the two places did not differ by more than the above quantity. The discharge at Helena, at the climax of the May flood, at the maximum stage of 38.75 ft., seems to have been about 950 000 cu. ft. ; and the Arkansas City discharge, four days later, at a height of 42.60 ft., should have been about 1 140 000 cu. ft. The difference between these quantities, or 190 000 cu. ft., should represent, within pretty narrow limits, the addition to the volume of the flood between Helena and Arkansas City.

In 1892, the Arkansas reached the tremendous height of 31.2 ft. at Little Rock on May 21st. The White River touched 30.4 ft. at Jacksonport. Now the Mississippi had passed its maximum at Helena by three weeks when this storm came, and had fallen 1.3 ft.; and the subsequent rise, which was due to the storm, reached the mouth of White River about June 16th, when it encountered the end of another rise, also of very formidable proportions.

But though it should happen that the crest of the rise of the main river should meet a tributary while at its greatest height, yet the effect upon the Mississippi will not be equal to that due to the maximum discharge of the tributary, unless the latter keep up for a month or so.

Topography of White River Basin.—What is commonly known as the White River Basin does not represent the whole reservoir surface available for the impounding of the waters of the two tributaries while in flood. The White River itself has a wide bottom, averaging,

perhaps, 4 or 5 miles in breadth, and it overflows its banks very readily. At Clarendon, it overflows at a stage of about 24 ft., some 10 ft. below high water. The Arkansas has a still wider and deeper bottom from Pine Bluff down, some 10 miles in breadth. The combined area of the basin proper and these outliers is about 1 160 square miles. When the Mississippi is high, the entire basin is filled, either from the main river or the tributaries, or both, the back water extending a long distance up both the minor streams. When the Mississippi is about bank-full and the tributaries very high, the latter still overflow their banks, especially White River, which is a stream of flat slope in its lower part. Of the main basin some parts are very low, far below mean high water or bank-full of the Mississippi, and such regions are overflowed in all considerable freshets of the subsidiary streams, which usually coincide with tolerably or exceedingly high stages of the Mississippi.

Effect of the Basin as a Reservoir.—If the maximum discharge of the Arkansas and White Rivers combined for several days is assumed to be 500 000 cu. ft., and if this volume is supposed to be poured into the White River Basin, already overflowed from the main river, it will inundate it in one day to the depth of 1.33 ft. If the discharge of the Mississippi is assumed to be increased at the rate of 50 000 cu. ft. for each foot of rise, the rise of 1.33 ft. will increase the discharge by about 66 500 cu. ft. in the first 24 hours, or for the average of the day, 33 250 cu. ft. This increased discharge will have the effect of lowering the water in the basin about 0.09 ft. The net rise in the basin in the first 24 hours will be 1.24 ft.

On the second day there will enter the basin 500 000 cu. ft. per second as before, but there will leave the basin 62 000 cu. ft., making the net gain 438 000 cu. ft. This will raise the basin about 1.17 ft. more; but this rise will cause an additional discharge of 58 500 cu. ft., or for the average of the second day, 29 250 cu. ft. This will cause a fall of about 0.08 ft.; net rise of the basin, 1.09 ft.; total rise, 2.33 ft. Third day, net discharge into the basin, 383 500 cu. ft.; rise, 1.02 ft.; additional mean discharge, 25 000 cu. ft.; consequent fall, 0.07 ft.; net rise, 0.95 ft.; total rise, 3.28 ft. A continuation of this process gives for the fourth day: Total rise, 4.11 ft.; fifth day, 4.88 ft.; sixth day, 5.52 ft.; seventh day, 6.06 ft. On the ninth day the rise of the basin would be about 7 ft.; on the twelfth about 8 ft.; on the seventeenth about 9 ft.; in a month, about 9.8 ft.

The freshets of the White and Arkansas Rivers do not remain long at an excessive height. In the great flood of 1892, the Arkansas remained at an extreme stage of 30 ft. or more at Little Rock for five days. Its extreme discharge of 400 000 cu. ft., or whatever it was, was for only one day. The stage of 30 ft. at Little Rock seems to correspond to a discharge of about 300 000 cu. ft. The water fell in two more days to 28 ft., and in two more to 26 ft., indicating discharges of 260 000 cu. ft. and 220 000 cu. ft. respectively. At Clarendon, the high stage was maintained with greater persistency from the effect of back water, but the discharge decreased from the falling off of the supply from above, and, consequently, the diminished slope.

Now, the rapid decrease of the supply into the reservoir very soon equalizes the inflow and the outflow, and presently the latter becomes the greater.

It must not be forgotten, however, that the tributaries sometimes remain for weeks together at stages much greater than the average. In 1890, White River remained for four months at an average stage of about 26 ft. at Jacksonport, and was never below 21 ft. In such cases the actual discharge produces its full effect, or nearly so.

Estimation of Maximum Discharge.—In estimating, then, the maximum discharge which has passed down the Mississippi in a given year at Helena, it will not be proper to take the maximum discharge at Cairo and add to it the maximum discharge of the Saint Francis; and for Arkansas City, to increase this by the maximum discharge of the White and Arkansas. Such a process, in 1882, resulted in absurd overestimates; and, indeed, the discharge of that year has always been exaggerated. As it is a matter of great consequence to obtain correct information on this point, a revision of the figures which have been presented will be made.

At Columbus.—A probable value for the Columbus discharge was found from the observations of that year to be 1 573 000 cu. ft. To this quantity a correction of 2% should be applied for the defective formula, bringing the net discharge to about 1 540 000 cu. ft. This corresponds with the results of the Columbus and New Madrid observations of 1893, quoted on page 467. Besides this, there was an escape over the banks, estimated at 86 000 cu. ft. Reasons have been given for doubting or altogether discrediting these estimates, and the calculation on page 481, made from the New Madrid measurements of 1893, of 1 600 000 cu. ft. for the total discharge of 1882 at Columbus, including what

passed outside of the channel, seems entirely reasonable. It may be affirmed that the measurements of 1893, checked at two points, made with the best instruments, by experienced men, and with the precautions shown to be necessary, and agreeing well together, taken, moreover, near extreme flood, are safe and trustworthy sources of information.

At Arkansas City.—The White River was at a stage of 21.8 ft. at Jacksonport when the crest of the rise passed the mouths of the two rivers; the Arkansas marked 15.1 ft. at Little Rock, and both were stationary, after having been much higher. It is likely, therefore, that they contributed nearly their full discharge, which was probably about 140 000 cu. ft. If 50 000 cu. ft. is added for the discharge of the Saint Francis, there will be about 1 790 000 cu. ft. for the volume which would have passed Arkansas City in a second in 1882 in a confined river. This estimate, however, is subject to deductions from several causes.

Deductions from Above Estimate.—The channel and flood plain of the river afford a considerable reservoir surface, to fill which will consume a portion of the discharge. This effect is particularly marked, as has been observed by Humphreys and Abbot,* toward the upper part of the alluvial valley, where the oscillations of height at the several gauge stations are sharper and more frequent. The effect, however, is cumulative, and the diminution of discharge continues as the river is descended. The extent of the reservoir influence will depend mainly on the duration of the rise. It is believed that an undue importance was attached to it by Humphreys and Abbot, who, reasoning from the experience of a short rise in 1858, estimated a regular loss of 140 000 cu. ft. of discharge between Cairo and Helena. Such a loss may occur where a heavy and sharp flood comes out on a low river and is succeeded by a rapid decline, but it cannot take place when the channel is already full and when the highest stage of the rise is prolonged for several days. Even under these circumstances, however, the reservoir influence is very perceptible. At Columbus, in 1882, the high stage of 102.8 ft. lasted only for a single day. In four days more the river had fallen to 102.1 ft., and the discharge presumably from 1 600 000 to 1 560 000 cu. ft. Now it takes about four days in a confined river for the crest of the rise to pass from Columbus to Helena. Taking the case of a confined river, and assuming the distance between levees when completed to be the same as already exists between Arkansas City and Wilson's

* "Report on the Physics and Hydraulics of the Mississippi River," p. 371.

Point, namely, about 2.5 miles, the reservoir space at high water between Columbus and Helena will be about 712 square miles, or about 19 850 000 000 sq. ft. The rise at Helena in four days was about 1 ft. The river at Columbus will be supposed to be stationary (it varied from 102.62 ft. to 102.79 ft.). To fill the basin an average of 0.5 ft. in one day would take about 10 000 000 000 cu. ft., or in four days about 2 500 000 000 cu. ft. This is equivalent to about 28 900 cu. ft. per second. This much of the discharge, therefore, will be consumed in filling the reservoir during the process of raising the river at Helena 1 ft., and the discharge which passes Helena will be less than the discharge which has passed Columbus by so much. On the fifth day, the discharge will be reduced by the fall from above by about the same amount (from about 1 600 000 to 1 571 500 cu. ft.), and the next day by as much more. As an approximation, the maximum discharge passing Helena in 1882, in a confined river, would have been nearly 30 000 cu. ft. per second less than passed Columbus. At Arkansas City, by the same mode of reasoning, the loss would have been increased to about 42 000 cu. ft., at Wilson's Point to 52 000 cu. ft.

Experience with confined waters at time of extreme flood is very limited. That condition is usually soon disturbed by crevasses and other complications. It is a time of great activity, anxiety and worry, and accurate observations of the behavior of the river are scarce. With such unknown conditions, it is possible that unexpected phenomena may be developed, causing an increased discharge to pass at very high stages. From causes which are already known, it is likely that flood heights at such stages will be diminished still further below the estimates given.

First.—No account has been taken of the discharge over banks or between the margin of the river on either side and the adjacent levee. The average distance between levees, as has already been remarked, from Arkansas City and Wilson's Point, is about 2.5 miles, of which space the river occupies nearly one-third. The levees are something like 12 ft. high. As the flood water gets deeper over these "foreshores," its velocity will increase rapidly. Even now, the "over-bank discharge" is an element which must be taken into account in every set of high-water measurements. At Arkansas City, in 1893, at the highest stage, this was estimated at 9 000 cu. ft., and at Wilson's Point it was over 30 000 cu. ft. Of course, the influence of this element will not be confined to the local space at each point that may lie between the

levee and the river, but will depend on the general section or the controlling section, whatever it may be.

Second.—At high stages the river flows between "water banks," as is often expressively said, for the upper part of its section, namely, that which is over bank-full height. Thus the friction and local disturbance ought to be somewhat diminished. It is not easy to estimate accurately the value of this element, but it would probably be not very great. In the ordinary formula, the substitution of banks of water for banks of earth for the upper 10 ft. of the section, even supposing the "water banks" to exert no resistance at all, would make a difference of only about 0.02 ft. per second in the velocity, or say 5 000 cu. ft. in the discharge.

Third.—It has been seen that it is possible for serious permanent alterations to occur in the bed of the river, and that the slope also undergoes modifications. This simply means that the river is relatively lower at some points than it used to be, as compared with the standard, Arkansas City. The confinement of the river, more or less complete, might be expected, indeed, to produce some effect on the bed. If a deterioration of the latter was brought about by the loss of volume over the banks, then it might reasonably be hoped that the retention of the water formerly lost by dispersion would have the effect of lowering the bottom in those places, so as to conform more nearly to the high-water slope.

Very sanguine expectations have been entertained of the scouring action of the powerful high-water forces thus brought into play. As the river never has been completely restrained, it cannot be said that the experiment has yet been fairly tried. The enlargement of cross-section at Wilson's Point, previously noticed, seems to have occurred in one season, from 1890 to 1891, and it has changed but little since that time. The relation between the gauges at Arkansas City and Lake Providence appears to have undergone no considerable alteration since 1885. The section at Arkansas City, with all its temporary mutations, remains the same. The discharge does not pass at a materially lower height than might have been predicted from the experience of previous years. So far, in the part of the river considered, the hopes entertained have not been realized.

Revised Estimate.—For all these causes, it seems reasonable to suppose that the extreme discharge of 1 790 000 cu. ft. found for Arkansas City in 1882 would have been diminished by at least 55 000 cu. ft.,

perhaps more, and that the maximum actual discharge which would have passed in a confined stream in that year would have been not greater than 1 735 000 cu. ft. This would require, by the methods previously used, a gauge height of about 55.8 ft.

It will be well to compare the floods of more recent years, and see whether the high water of 1882 was really the greatest of which there is any record, and consequently whether it was correctly adopted as a standard.

In 1883, the river attained its greatest recorded height at Cairo. There was a considerable flood in the Arkansas and White Rivers, but it did not coincide with the crest of the main rise. The river below Arkansas City was at an unusually depressed stage when the rise began; so the reservoir capacity of the channel was very great. Consequently the flood wave flattened out, and the gauge at Vicksburg was 5 ft. lower than in 1882.

The flood of 1884 was one of the greatest ever known. It may fairly be compared with 1882. The gauge at Cairo and at Helena was nearly the same as in 1882. At Vicksburg it was 0.15 ft. higher. No material change had occurred in the situation. No discharges were taken in 1884, and it is not easy to institute minute comparisons between them.

In 1886, there was a short and sharp rise which reached 51 ft. at Cairo and 48.1 ft. at Helena, the levees of the upper Yazoo District having been recently rebuilt. There were no serious freshets from the White and Arkansas Rivers, and the river at Vicksburg was 4.6 ft. lower than in 1882.

There was a great flood in 1890. It was not very high at Cairo, only 48.8 ft., but it was very long and persistent, and it was accompanied by an unparalleled stage of White River, which stood for four months at a mean height of 26 ft. at Jacksonport, the highest being 33.2 ft. (the greatest on record), and the lowest 20 ft. The Arkansas was not so high, but was above medium stage most of the time. There were two great freshets of the tributaries, the former and greater of which occurred before the Mississippi had reached its height. The second took place in April, after the main rise, but during the later rise which succeeded the first, which was nearly as high. The mean of twelve consecutive days at Helena gives a discharge of about 1 470 000 cu. ft. (reduced) at an average stage of 47.24 ft., with a crevasse 19 miles above, discharging about 30 000 cu. ft. The

Arkansas and White Rivers no doubt added some 200 000 cu. ft. to this volume, making in all about 1 700 000 cu. ft. There were many breaks in the levees on both sides of the river.

The flood of 1892 reached a height at Cairo of only 48.3 ft., and remained there only a short time. The year would not have been remarkable had it not been for the tremendous outpour of the Arkansas, accompanied by a very high stage of the White River, shortly before alluded to, which made it extremely formidable, and would indeed have made it disastrous to the levees had it not been for the escape of a large part of the waters of the Arkansas around the head of the Tensas Basin system. The discharge at Columbus is given as about 1 380 000 cu. ft. for a mean of five days, but as the velocities were taken at 0.6 depth, a correction should be applied, which, at Columbus, would amount to 5% or so. At Helena, for six days, the discharge is reported as 1 302 000 cu. ft., but as the meter had not been rated during the season, the results here and at Columbus cannot be relied on. The Arkansas City discharges taken at and after the height of the rise seem to be free from the disturbing influences which had caused so much trouble earlier in the season, and are coherent and consistent with the measurements of the succeeding year. For a mean of ten days at the top of the flood, the discharge is given as about 1 450 000 cu. ft.; corrected for 0.6 depth, by the factor habitually used in 1893 at Arkansas City, about 1 425 000 cu. ft. The overflow around the end of the levee and through breaks along the Arkansas River was estimated at about 280 000 cu. ft. These estimates are always too great. Doubtless the total discharge was between 1 650 000 and 1 700 000 cu. ft.

The flood of 1893 has been pretty thoroughly discussed. The total discharge passing Arkansas City would no doubt have been about 1 600 000 cu. ft.

It is now possible to answer the question propounded on page 475, which was the cause of the discussion of the effect of reservoirs on the discharges of tributaries, namely, whether the increased gauge height produced at Helena by the confinement of the river would be propagated down stream. In 1882, at Arkansas City, the height would have been about 55.8 ft., and the discharge of the two tributaries would have increased the gauge height at Arkansas City about 3 ft. over that of Helena. It was thought probable that the latter would have been, in a confined river, about 53 ft. (see pages 475 and 480). The conclu-

sion is that the increased gauge height at Helena would extend its influence to the other gauges further down stream.

These inferences might be substantiated by reasoning drawn from the relations between the gauges. This part of the subject, however, has already been well and thoroughly discussed by others.

Low-Water Discharge.

Comparatively little attention has been paid to the low-water discharge. Very few observations have been taken at that stage. The least discharge recorded in the reports of the Mississippi River Commission was in the fall of 1891. In November of that year, at a gauge reading of 2.9 ft. at Cairo, the discharge was reported as 114 203 cu. ft. At Elmot Landing, in Plum Point Reach, near the same time, the mean of the four lowest days was about 81 400 cu. ft. At Memphis, at an equally low stage in October, it was reported as about 91 000 cu. ft. At Helena, the lowest record, both in October and in November, for the mean of several days, was about 106 000 cu. ft. at a stage of zero. At Wilson's Point, in November, there was a minimum discharge reported (one observation only) of 117 142 cu. ft. At Red River Landing, the lowest record was 141 179 cu. ft.

The observations of the last four years have shown a perceptible depression of the low-water plane at Wilson's Point, as will be seen by the following table:

Year.	Gauge.	Discharge.
1891	5.00 ft.	133 000 cu ft.
1892	4.7 "	145 000 "
1893	2.1 "	156 000 "
1894	2.3 "	177 000 "

In 1894, this depression was general, and extended at least from Memphis to Vicksburg, the low-water record having been broken at every intermediate point. That this was not due to the small quantity of water flowing in the river is shown by the discharge at Wilson's Point, which was greater than in 1891. What significance is to be attached to this phenomenon is not yet known, and probably further observation will be required to develop it. It may not be permanent. It is to be observed that the stage in 1894 was very nearly paralleled in 1872 at Memphis and at Lake Providence for a day or two. The low water of 1894, however, was far more persistent, the gauge at Vicksburg remaining below zero for nearly three months.